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Short Communication

Numerical Simulation of Pore Water Distribution in Unsaturated Soils under Electric Field

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A numerical model for predicting the moisture distribution under an external electric field in unsaturated soils is established. Key parameters, including matric potential, hydraulic and electrical permeability coefficients and electric conductivity, are discussed. The simulation results show agreement with the measured data from the literature, which verifies the exactness and suitability of the model. In general, the moisture contents decrease with time under the action of an external electric force and matrix suction. A slight increase in moisture content was observed in the anodic area at the beginning of the treatment. The electro-osmosis treatment effect can be enhanced by improving the voltage gradient or weakening the voltage loss at the electrode. The limit value of electroosmosis treatment in moisture content is observed, and the method can only be applied to the soil with a moisture content that exceeds the limit value.

Keywords: Unsaturated soils, water distribution, governing equation, hydraulic permeability coefficient

1. INTRODUCTION

The pore water flow generated during the electro-osmosis (EO) process is irrelevant to the hydraulic conductivity. Therefore, electro-osmosis treatment on fine-grained soil with low permeability has been applied to an increasing number of practical geotechnical projects [1,2], including disposals of industrial sludge, marine clay reinforcement and stabilization, land reclamation, foundation excavation and so on. To precisely understand the EO process and to provide guidelines for project practice, critical parameters [3,4], such as electrode material, applied voltage gradient, soil conductivity and additional chemical treatment, were analyzed through indoor experiments, and abundant theoretical models were investigated. One-dimensional theory was first derived by Esrig [5], which assumes that water flow generated by hydraulic forces and electric forces can be linearly superimposed. Wan and Mitchell [6] developed a two-dimensional model that takes into account the coupled effects of surcharge and EO.

Most of the existing electroosmotic theories are focused on the consolidation of saturated soil, which fails to consider unsaturated soil. In addition to unsaturated soils being less common in coastal areas where electroosmosis is often used to treat dredged soft soils, the consolidation theory of unsaturated soil involves interactions among solid, liquid and gas phases. The governing equations are usually highly nonlinear, and it is difficult to obtain an analytical solution. On the other hand, the volume of H_2 and O_2 generated by the electrolysis reaction at the electrode that intruded the soil, leading to a desaturation of the soil, is hard to quantify. Researchers used finite element software to simulate the electroosmosis of unsaturated soil [7,8], assuming that only the O_2 generated by the anode entered the soil and that H_2 at the cathode was expelled directly into the atmosphere. The ratio of the gas entering the soil to the total generated oxygen is a constant value η. The simulation results show that the value of η has a strong effect on the development of pore pressure. For instance, the final pore pressure in the anode soil reaches -200 kPa when η=0.1, while it reaches -500 kPa when η=0.4.

The behavior of unsaturated soil under an electric field is quite different from that of saturated soil. In this paper, a numerical model for unsaturated soil water distribution under the electroosmosis process is established. The simulation results are compared with the existing data in the literature.

2. NUMERICAL MODEL

2.1 Governing equation

The model is one-dimensional; thus, the directions of current, flow and settlement are vertical and parallel to each other, as shown in Fig. 1. The assumptions for the model are listed here.

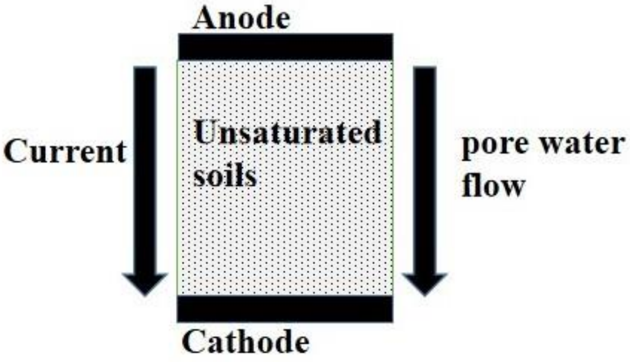

Figure 1. Schematic of one-dimensional electroosmosis of unsaturated soil

1) The migration of pore water follows Darcy's law.

2) Gases generated by hydrolysis reactions enter the atmosphere, and the pores in soil are connected with the atmosphere.

3) The hydraulic permeability, electric permeability coefficient and electrical conductivity are functions of moisture content.

4) The soil shape does not change with the change in water content.

5) Other assumptions are similar to those of the Esrig one-dimensional electroosmotic consolidation theory.

The seepage of unsaturated soil usually considers the total potential energy of pore water as the variable. The flow rate caused by the gravity potential and matrix potential can be written as:

$$\vec{q}_h = -k_h(\theta)\cdot\nabla\psi \tag{1}$$

$$\psi = \psi_m(\theta) + z \tag{2}$$

In the equations above, $k_h(\theta)$(m/s) is the hydraulic permeability coefficient of unsaturated soil. Ψ(m) is the soil water potential, $\Psi_{m}(\theta)$(m) represents the matrix potential, and z(m) denotes the gravity potential. θ is the volumetric moisture content, which can be linearly converted to mass moisture content and is adopted in this paper.

The flow rate under an electric field can be described as follows:

$$\vec{q}_e = -k_e(\theta)\cdot\nabla\phi \tag{3}$$

where $k_e(\theta)$ is the electric permeability coefficient of unsaturated soil and ϕ is the electric potential. According to Esrig's theory, seepage caused by an electric field and gravity potential can be superimposed linearly. Hence, the total flow rate q is:

$$\vec{q} = \vec{q}_e + \vec{q}_h = -k_e(\theta)\cdot\nabla\phi - k_h(\theta)\nabla\psi \tag{4}$$

In this situation, disregarding the mass of water electrolysed and assuming pore water does not move soil particles when it migrates, the mass continuity equation for the liquid phase can be written as:

$$\frac{\partial\theta}{\partial t} + \nabla\cdot\vec{q} = \frac{\partial\theta}{\partial t} + \nabla\cdot[-k_e(\theta)\cdot\nabla\phi - k_h(\theta)\nabla\psi] = 0 \tag{5}$$

For an isolated system consisting of soil and electric circuits, the total charge is conserved. Ohm's law and the principle of charge conservation give us the following equation:

$$C_p\frac{\partial\phi}{\partial t} + \nabla\cdot[-\sigma_e(\theta)\nabla\phi] = 0 \tag{6}$$

where C_p(F/m^3) is the unit capacitance and $\sigma_e(\theta)$(s/m) is the conductivity of unsaturated soil. Equations (5) and (6) are the governing equations for the coupled electric-seepage field of unsaturated soil.

2.2 Soil characteristic parameters

1) The matric potential $\Psi m(\theta)$

The matrix potential of soil can be described by the soil water characteristic curve; a typical curve is illustrated in Fig. 2. The inflection point h_d is the critical value of air-entry suction, which is related to the particle size, pore size and shape of the soil particles.

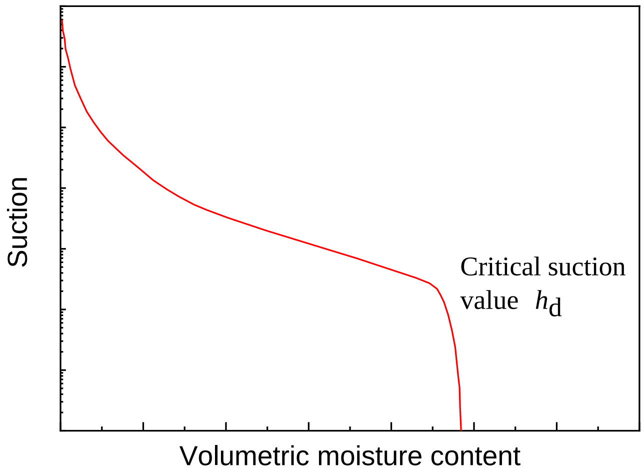

Figure 2. Typical soil water characteristic curve

To describe the mathematical form of the curve, the standardized volumetric moisture content Θ and effective saturation S_e are adopted as follows:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{7}$$

$$S_e = \frac{S - S_r}{1 - S_r} \tag{8}$$

θ_r and S_r are the residual volume moisture content and residual saturation, corresponding to only hydroscopic water existing in soils. θ_s is the saturated volume moisture content. It is notable that these two indices are numerically equal. This paper adopts an empirical model established by Van Genuchten [9] to characterize the curve. The expression is presented as follows:

$$\Theta = S_e = \left[\frac{1}{1 + (a\psi_m)^n}\right]^m \tag{9}$$

where a --- fitting parameter related to air intrusion,

m --- fitting parameter of the curve, $m=1-1/n$

n --- parameter related to the pore distribution of soil

2) Hydraulic permeability coefficient of unsaturated soil

Calculating the permeability coefficient of unsaturated soil from conventional constitutive equations, such as the soil-water characteristic curve, or making predictions from experimental data is concerning to many scholars. Mualem [10] classified these methods into three categories: empirical models, macroscopic models and statistical models. In this paper, a statistical model proposed by Van Genuchten is employed. The model comprehensively considered the soil-water characteristic curve and obtained a closed smooth equation:

$$k_{h,rel}=\left[\frac{1-(\alpha\psi_m)^{n-1}\left[1+(\alpha\psi_m)^n\right]^{-m}}{\left[1+(\alpha\psi_m)^n\right]^{m/2}}\right] \tag{10}$$

Submitting the standardized volume moisture content into Equation (10), we obtain:

$$\begin{cases} k_{h,rel}=\Theta^{1/2}\left[1-\left(1-\Theta^{1/m}\right)^m\right]^2 \\ k_{h,rel}=k_h/k_{h,sat} \end{cases} \tag{11}$$

where $k_{h,rel}$--- relative hydraulic permeability coefficient

$k_{h,sat}$--- hydraulic permeability coefficient when saturated

3) Electrical permeability coefficient of unsaturated soil

The water content decreases with the development of electroosmosis, leading to desaturation among the soil. The decrease in saturation will cause an increase in resistivity and a decrease in the permeability coefficient. For saturated soil, Helmholtz-Smoluchowski (H-S) theory has been widely accepted in current research on the coefficient of electrical permeability. This model is established on the basis that the electric field forces applied to the ions balance the viscous forces due to uneven water velocity in a stable flow, which fails to consider the effect of matrix suction in regard to unsaturated conditions. Studies show that in fine-grained soils, matric suction at low saturation has a greater effect than that at other water heads [11]. A modified model [12] that takes into account the degree of saturation is needed in this paper. The relative electroosmotic permeability can be expressed using a power function:

$$\begin{cases} k_{e,rel}=a(S)^b \\ k_{e,rel}=k_e/k_{e,sat} \end{cases} \tag{12}$$

Submitting the standardized volume moisture content into Equation (11), we obtain:

$$\begin{cases} k_{e,rel}=a[S_r+\Theta(1-S_r)]^b \\ k_{e,rel}=k_e/k_{e,sat} \end{cases} \tag{13}$$

where $k_{e,rel}$--- relative electroosmotic permeability coefficient

$k_{e,sat}$--- electroosmotic permeability coefficient when saturated

a, b--- fitting parameters

4) Soil conductivity

Soil conductivity is influenced by moisture content, fluid salinity, temperature and many other factors. Under the conditions of low water content (10%~60%) and constant porosity, the conductivity shows an approximate linear increase with water content [13]. According to the assumption above, the geometric deformation is disregarded, and the moisture content for unsaturated soil is relatively low. Hence, the conductivity of the soil is:

$$\sigma_e(\theta)=A+B\theta \tag{14}$$

where $A=-0.027$ and $B=32$.

3. RESULTS AND DISCUSSION

3.1. Testing parameters

To validate the rationality and validity of the model, an experiment [14,15] was compared with the results of the model. The soil in the literature has a moisture content in the range of 20.3~29.8%, liquid limit of 38.69% and plastic limit of 22.16%. The soil-water characteristic curve fitting parameters are listed in Table 1.

Table 1. Soil-water characteristic curve fitting parameters

θ_s(%)	θ_r(%)	S_r	α(kPa^{-1})	Fitting parameter *m*	Fitting parameter *n*
43.96	29.24	0.665	0.002	1.79	1.86

The standardized volume moisture content is:

$$\Theta = \frac{S - S_r}{1 - S_r} = \left[\frac{1}{1 + (\alpha\psi_m)^n}\right]^m = \frac{\theta - 29.24}{43.96 - 29.24} = \left[\frac{1}{1 + (0.002\psi_m)^{1.86}}\right]^{1.79} \qquad (15)$$

The permeability coefficient of saturated soil, which was measured in the laboratory, is $k_{h,sat}=2.31\times10^{-9}$ m/s, and the hydraulic permeability coefficient for unsaturated soil is:

$$\begin{aligned} k_h &= k_{h,sat}\Theta^{1/2}\left[1-\left(1-\Theta^{1/m}\right)^m\right]^2 \\ &= 2.31\times10^{-9}\times\left(\frac{\theta-29.24}{43.96-29.24}\right)^{0.5}\times\left\{1-\left[1-\left(\frac{\theta-29.24}{43.96-29.24}\right)^{\frac{1}{1.79}}\right]^{1.79}\right\}^2 \end{aligned} \qquad (16)$$

The typical electric permeability coefficient [16] of saturated soil is 2×10^{-9} $m^2/s\cdot V$, and the fitting parameters are *a=1 and b=3*. Therefore, the electrical permeability coefficient of unsaturated soil is:

$$k_e = 2.0\times10^{-9}\times\left[0.2924 + \frac{\theta-29.24}{43.96-29.24}(1-0.2924)\right]^3 \qquad (17)$$

3.2. Boundary condition

The experimental diagram is shown in Fig. 3. A 300 mm height cylindrical soil sample, encompassed by a ceramic column with measuring holes, is sandwiched by the anode above and the cathode beneath. The applied voltage is 30 V, and the gradient is 1 V/cm. The z-axis takes the bottom of the soil sample as the origin, and the vertical upward direction is positive. The experiment starts after the capillary water reaches a stable state.

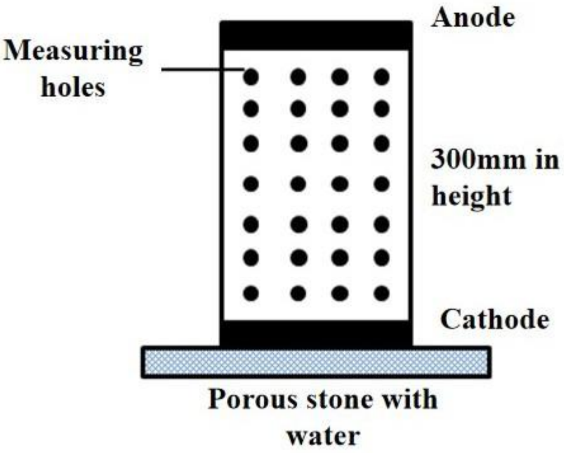

Figure 3. Diagram of one-dimensional electro-osmosis experiment

The initial water distribution and its nonlinear fitting equation are given by the literature above:

$$\theta\Big|_{t=0} = 44.56 - 14.58e^{-0.5\times\left(\frac{z-0.2809}{0.0943}\right)^2} \tag{18}$$

The boundary conditions are listed as follows:

$$\begin{cases} \left.\dfrac{\partial\theta}{\partial z}\right|_{z=0} = k_{h,sat} = 2.31\times10^{-9} \\ \left.\dfrac{\partial\theta}{\partial z}\right|_{z=0.3} = 0 \\ \phi_{|z=0} = 0 \\ \phi_{|z=0.3} = 30 \end{cases} \tag{19}$$

3.3. Results and discussion

The partial differential equations are solved using MATLAB. The measured data in the experiment and the calculation results of the above equations are presented here. Fig. 4 illustrates the volume moisture contents at different heights of soil at different times. Data from four separate locations, z=50 mm, 130 mm, 210 mm, and 250 mm, are investigated.

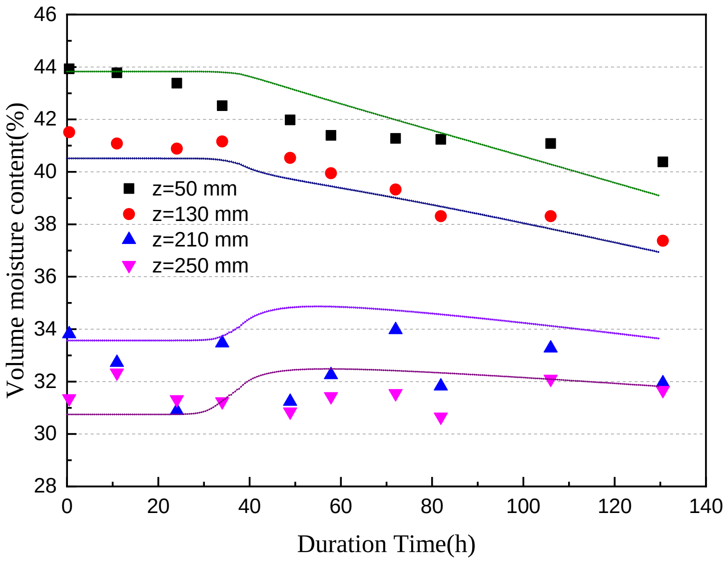

Figure 4. Volume moisture content distribution during 1V/cm vlotage gradient treating process

The simulated value coincides with the experimental results. At z=50 mm and 130 mm, the volume moisture content decreases with time. At z=210 mm and 250 mm, the water content declines gradually after a slight increase.

Pore water migration is driven by both electric force and matric suction; the suction of the soil matrix causes upward migration, while electric force causes downward migration. When the moisture content is high, i.e., when z=50 mm and 130 mm, the matrix suction is smaller than the electric force, which causes moisture to decrease with time. In contrast, matrix suction dominates the migration trend when the water content is relatively low at z=210 mm and 250 mm at the beginning of the test. After these two forces reach equilibrium at approximately 35 h, the pore water reverses its migration direction. The above results indicate that the effect of electroosmosis is more suitable for unsaturated soil with a high water content rather than those with a low water content, which is similar to that of saturated soil.

After a sufficiently long treatment time, it is determined that the final value throughout the soil tends to remain stable; in this case, approximately 29~30% at 1.7×10^5 h (refer to Fig. 5). Soil with a high initial moisture content decreases at a faster rate, while it decreases at a slower rate with a relatively low initial moisture content. Under ideal conditions, the water will move to the cathode where drainage is allowed under the action of matrix suction and electric force when an external electric field is applied. When the water content drops to a certain critical value, the effect of the matrix potential and electric potential on the pore water reaches a dynamic balance, indicating that the water content no longer changes over time. This finding indicates that there exists a limit in electro-osmosis treatment: if the electroosmosis method is employed to treat the soil with a water content lower than the critical value, the effect cannot be achieved.

From the above discussion, the critical value can be reduced by increasing the electric field force or by decreasing the matric suction. Generally, it is easier to improve the electric field force, for instance, by improving the potential gradient [17] or by employing electrode materials with low potential loss. It has been reported that the average anode contact resistance in electrokinetic geosynthetic electrodes is 56% smaller than that of iron electrodes [18].

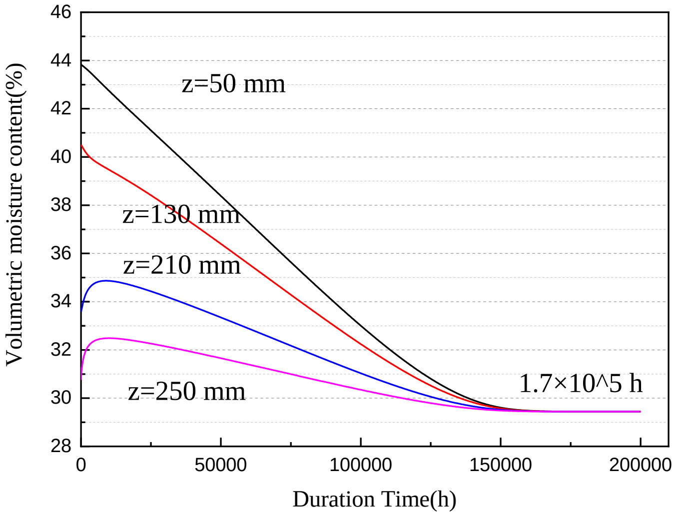

Figure 5. Volume moisture content distribution after 1V/cm voltage gradient treatment

4. CONCLUSION

In this paper, the governing equation for pore water distribution under an electric field in unsaturated soils is driven through the mass continuity equation and principle of charge conservation. From the statements above, we can conclude the following information:

1) The simulation results are highly consistent with the data in the literature, which validates the applicability of the one-dimensional water migration model.

2) Numerical results show that the moisture contents decrease with time under the action of external electric force and matrix suction. A slight increase in the moisture concentration was observed in the anodic area at the early stage due to the matrix potential. The electro-osmosis treatment method can only be applied to soil with moisture contents higher than the critical value.

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References

1. N.C. Lockhart, *Colloids Surf.*, 6(1983)229.
2. J.K. Mitchell, *Geotechnique,* 41(1991)299.
3. X.Y. Xie, L.W. Zheng and K.H. Xie. *Chin. Civ. Eng. J.*, 52(2019)108
4. C.Y.Ou, S.C. Chien, C.C. Yang and C.T. Chen, *Appl. Clay Sci.*,104(2015)135

5. M.I. Esrig, J.SMFD, ASCE. 94(1968)899.
6. T.Y. Wan and J.K. Mitchell, *J. Geotech. Eng*. D., 102(1976)473.
7. C. Tamagnini, C. Jommi and F. Cattaneo. *Ana. D. Ac. Bra. Ci.*, 82(2010)169.
8. J. Yuan and A.H. Michael. *Int. J. Numer. Anal. Methods Geomech*., 40(2016)1570.
9. M.T. Van-Genuchten , *Soil Sci. Soc. Am. J.*, 44(1980)892.
10. A. Klute. Hydraulic conductivity of unsaturated soils: Prediction and formulas .Methods of Soil Analysis, Madison, 1(1986)799.
11. L. Ning and J.L. William. *J. Geotech. Geoenviron. Eng.*, 132(2006)131.
12. X.Y. Xie, Y.M. Liu and L.W. Zheng. *Mar. Georesour. Geotechnol.*, 37(2019)1188
13. Y. Li, X.N. Gong, B. Guo and Z.G. Zhang. Chin. *J. Rock Mech. Eng*., 29(2010)4027.
14. H.Y. Zhang, Y.X. Gai, S.B. Zhu and Y.M. Sheng. *J. Lanzhou Univ. Nat. Sci.*, 52(2016)571.
15. B.Z. Yang. Moisture redistribution in the unsaturated soil subjected to DC electric field. Lanzhou Univ., 2017.
16. J.Q. Shang. *Can. Geotech. J.* 34(1997)627.
17. J.Q. Shang, K.Y. Lo and K.M. Huang. Effects of voltage gradient and polarity reversal on electro-osmotic consolidation. Proc. 2nd Int. Conf. Soft Soil Eng., (1996)966.
18. J.C. Zang., L.W. Zheng and X.Y. Xie. *J. Cent. South Univ*, 25(2018)3052.