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# Analytical Expression of Concentrations and Current in Enzyme-Based Two-Compartment Model of Amperometric Biosensors for Steady-State Condition

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In this paper, the two-compartment mathematical model of the amperometric biosensor for the steady-state condition is studied. This model relies on a nonlinear reaction-diffusion system with Michaelis-Menten kinetics. A novel new approach of the homotopy perturbation method (HPM) is used in the enzyme and membrane layer to solve nonlinear reaction-diffusion equations. The simple and closed-form of analytical expressions of concentrations and biosensor current is derived. This paper presents the approximate analytical expression of sensitivity and resistance of a two-compartment model of amperometric biosensors. Furthermore, in this work, the numerical solution of the problem is also reported using the Maple program. The obtained results are compared with the numerical and previous available limiting case results. The analytical result provided is effective and precise in understanding the behaviour of the system.

**Keywords:** Mathematical modelling; Amperometric biosensor; Two-compartment model; Nonlinear reaction diffusion equation; Enzyme kinetics.

## 1. INTRODUCTION

Catalytic biosensors are analytical devices that depend on the enzyme-catalyzed conversion of active redox products [1]. Due to the limitations of free-diffusing redox compounds, particularly in terms of continuous monitoring of the analyte, the use of various semi-permeable membranes enables the design of dependable and susceptible electrocatalytic systems [2]. A thin layer of polyvinyl alcohol, terylene, cellulose, or another substance frequently uses in commercial biosensors to keep the enzyme from decomposing [3].

It is critical to predicting both geometric and catalytic parameters when solving analytical problems and developing new biosensors. Multi-layer models are essential for modelling biosensors with selective membranes to attain appropriate model accuracy [4,5]. Nonetheless, due to the model's simplicity, even monolayer models that disregard external mass movement by diffusion are still utilized in various applications [6]. Lyons [7] provided the analytical solutions for steady- state conditions for both the amperometric and potentiometric biosensor.

The two-compartment models and a type of multi-layer model have been used to model biosensors [5,8-12]. The two-compartment biosensor model is characterized by a relatively thin layer of an enzyme and an external diffusion layer [5]. Various desirable characteristics of biosensors have been identified using two-compartment models [10,12].

Baronas [8,13] solve this initial boundary value problem numerically. We used the new homotopy perturbation approach to construct an approximate analytical solution of concentration and current to describe and analyze the performance biosensor in this work.

## 2. MATHEMATICAL MODELLING

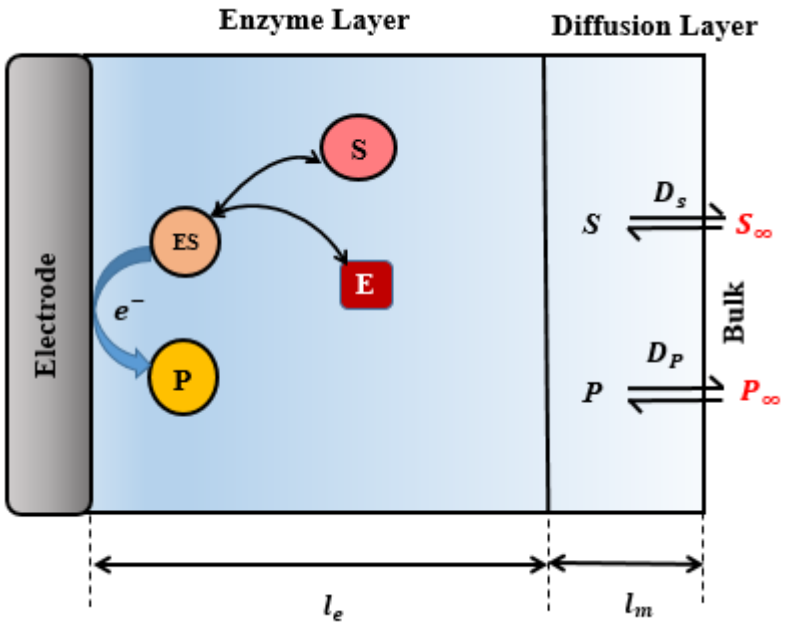


**Figure 1.** The schematic diagram represents an amperometric biosensor.

A principal form of the enzyme-catalyzed reaction is,

$$E + S \leftrightarrow ES \rightarrow E + P \quad (1)$$

where substrate is $S$, $E$ is enzyme and to form a complex ($ES$). After product P is dissociated, the complex regenerates the enzyme [15,16]. Because the intermediate complex (ES) does not change in a quasi-steady state, it is possible to model biosensors biochemical behaviour [17]. As a result, the substrate (S) is converted to the product (P).

$$S \rightarrow P \quad (2)$$

The electro-active product P is converted at the electrode surface into a species P' that has no effect on the biosensor's function,

$$P \rightarrow P' \pm e^{-} \quad (3)$$

The dimensionless form of the two-compartment model was obtained by rescaling space, concentrations, and diffusion coefficients as follows [14]:

$$\frac{d^2S_e}{dX^2} - \sigma^2 \frac{S_e}{1+S_e} = 0 \quad (4)$$

$$D_{Pe}\frac{d^2P_e}{dX^2} + \sigma^2 \frac{S_e}{1+S_e} = 0, X \in (0,1) \quad (5)$$

The governing equations are given as follows:

$$\frac{d^2S_m}{dX^2} = 0 \quad (6)$$

$$\frac{d^2P_m}{dX^2} = 0, X \in (1,1+L_m) \quad (7)$$

The boundary conditions are

$$P_e(0) = 0, \left.\frac{dS_e}{dX}\right|_{X=0} = 0 \quad (8)$$

$$S_m(1+L_m) = S_0, P_m(1+L_m) = 0 \quad (9)$$

$$\left.\frac{dS_e}{dX}\right|_{X=1} = D_{Sm}\left.\frac{dS_m}{dX}\right|_{X=1}, S_e(1) = S_m(1) \quad (10)$$

$$\left.\frac{dP_e}{dX}\right|_{X=1} = \frac{D_{Pm}}{D_{Pe}}\left.\frac{dP_m}{dX}\right|_{X=1}, P_e(1) = P_m(1) \quad (11)$$

where

$$S_e = \frac{s_e}{k_m}, P_e = \frac{p_e}{k_m}, S_m = \frac{s_m}{k_m}, P_m = \frac{P_m}{k_m} \quad X = \frac{x}{l_e}, L_m = \frac{l_m}{l_e}, K_M^{app} = \frac{k_M^{app}}{k_M}, \; S_0 = \frac{s_0}{k_m}$$

$$D_{Pe} = \frac{d_{Pe}}{d_{Se}}, D_{Sm} = \frac{d_{Sm}}{d_{Se}}, D_{Pm} = \frac{d_{Pm}}{d_{Se}}, \sigma^2 = \frac{v_{max}l_e^2}{k_m d_{Se}} \quad (12)$$

where $S_e, P_e, S_m, P_m$ is the dimensionless concentrations of the substrates and products in the enzyme layer and membrane layer, $D_{Se}, D_{Pe}, D_{Sm}, D_{Pm}$ are the dimensionless diffusion coefficients, $\sigma$ is the dimensionless diffusion module or the Damköhler number [5,13,18]. The dimensionless current '$I$' is as follows:

$$I = D_{Pe}\left.\frac{dP_e}{dX}\right|_{X=0} = \frac{il_e}{n_e F d S_e k_M} \quad (13)$$

## 3. ANALYTICAL EXPRESSION FOR THE CONCENTRATIONS OF THE SUBSTRATE ($S_e$ and $S_m$) AND THE PRODUCT ($P_e$ and $P_m$).

Mathematical models arising from modern, sophisticated biochemical reactions are frequently nonlinear differential systems with no concrete solutions. Although most numerical solutions are reasonably easy to find, researchers are continually concerned about flaws such as loss of stability of modifying parameters to match numerical data. Fortunately, in recent years, many trustworthy and highly accurate analytical procedures have been created using sophisticated computational tools.

Variation iteration [19], homotopy perturbation [20,21], differential transformation [22,23], Green's function iterative [24,25], exp-function [26], and series technique [27], Adomian decomposition [28] are just a few examples of these methods. In this communication, we foresee that the wider science community would escalate the application of highly accurate and efficient analytical methods (homotropy perturbation method) accessible to all.

Solving the equations (4) to (7) for the BC's (8) to (11), using new homotropy perturbation approach [29, 30] , the approximate analytical solution for concentrations of substrates ($S_e, S_m$) and the product ($P_e, P_m$) are obtained ( Appendix-A) as follows:

$$S_e(X) = \frac{S_0 D_{Sm} \cosh\sqrt{\mu} X}{L_m \sqrt{\mu} \sinh\sqrt{\mu} + D_{Sm} \cosh\sqrt{\mu}} \tag{14}$$

$$P_e(X) = \frac{S_0 D_{Sm} \left[ \frac{X(L_m D_{Pe} \sqrt{\mu} \sinh\sqrt{\mu} + D_{Pm} \cosh\sqrt{\mu} - 4D_{Pm})}{L_m D_{Pe} + D_{Pm}} - (\cosh(\sqrt{\mu} X) - 4) \right]}{D_{Pe}(L_m \sqrt{\mu} \sinh\sqrt{\mu} + D_{Sm} \cosh\sqrt{\mu})} \tag{15}$$

$$S_m(X) = \frac{S_0[(X-1)\sqrt{\mu} \sinh\sqrt{\mu} + D_{Sm} \cosh\sqrt{\mu}]}{L_m \sqrt{\mu} \sinh\sqrt{\mu} + D_{Sm} \cosh\sqrt{\mu}} \tag{16}$$

$$P_m(X) = \frac{S_0 D_{Sm}(1 + L_m - X)[\sqrt{\mu}\sinh(\sqrt{\mu}) - \cosh(\sqrt{\mu}) + 4]}{(L_m D_{Pe} + D_{Pm})(L_m \sqrt{\mu} \sinh\sqrt{\mu} + D_{Sm} \cosh\sqrt{\mu})} \tag{17}$$

where $\mu = \frac{\sigma^2}{1 + S_e(0)}$ and $S_e(0) = \frac{S_0 D_{Sm}}{L_m \sigma \sinh\sigma + D_{Sm} \cosh\sigma}$ (18)

The dimensionless currents,

$$I = \frac{i l_e}{n_e F d S_e k_M} = \frac{S_0 D_{Sm}(L_m D_{Pe} \sqrt{\mu} \sinh\sqrt{\mu} + D_{Pm}(\cosh\sqrt{\mu} - 4))}{(L_m D_{Pe} + D_{Pm})(L_m \sqrt{\mu} \sinh\sqrt{\mu} + D_{Sm} \cosh\sqrt{\mu})} \tag{19}$$

When $\sqrt{\mu}$ is very small, $\sinh\sqrt{\mu} \approx \sqrt{\mu}$ and $\cosh\sqrt{\mu} \approx 1$. In this case the dimensionless current becomes

$$I \approx \frac{S_0 D_{Sm}(L_m D_{Pe} \mu - 3D_{Pm})}{(L_m D_{Pe} + D_{Pm})(L_m \mu + D_{Sm})} \tag{20}$$

Recently Romas Baronas et al. [14] obtained current for the limiting cases as follows:

$$I = \frac{i l_e}{n_e F d S_e k_M} = \frac{S_0 D_{Pe}(D_{Sm} - \sigma^2)\,[L_m D_{Pe} \sigma \sinh(\sigma) + D_{Pm}(\cosh(\sigma) - 1)]}{(L_m D_{Pe} + D_{Pm})(L_m \sigma \sinh(\sigma) + D_{Sm} \cosh(\sigma))}, S_0 \ll 1 \tag{21}$$

$$I = \frac{i l_e}{n_e F d S_e k_M} = \sigma^2 \left( \frac{D_{Pm} + 2L_m D_{Pe}}{2(D_{Pm} + D_{Pe} L_m)} \right), S_0 \gg 1 \tag{22}$$

### *3.1 Sensitivity Of Biosensor*

The most significant characteristics of biosensors is sensitivity. A sensitivity of the biosensor $B_S$ can be described the maximum biosensor current density vs substrate concentration $S_0$ [31]. The dimensionless sensitivity is calculated as follows:

$$B_S(S_0) = \frac{\partial I(S_0)}{\partial S_0} \frac{S_0}{I(S_0)} = \frac{S_0 D_{Sm} L_m \sigma^2 (D_{Pe} D_{Sm} + 3D_{Pm})(L_m \sigma^2 + D_{Sm}) \left( \frac{L_m \sigma^2}{1 + \frac{S_0 D_{Sm}}{L_m \sigma^2 + D_{Sm}}} + D_{Sm} \right)}{\left( (S_0 + 1) D_{Sm}^2 + 2 D_{Sm} L_m \sigma^2 + L_m \sigma^4 \right)^2 \left( 3D_{Pm} - \frac{L_m \sigma^2}{1 + \frac{S_0 D_{Sm}}{L_m \sigma^2 + D_{Sm}}} \right)} \tag{23}$$

where $I(S_0)$ is the density of the steady-state biosensor current.

### *3.2 Resitance of Biosensor*

The resistance of biosensors $B_R$ is discussed. The biosensor current with respect to the enzyme layer thickness $L_m$ [31] is expressed as the dimensionless resistance of the biosensor.

$$B_R(L_m) = \frac{L_m}{I(L_m)}\frac{dI(L_m)}{dL_m} = \frac{(S_0 L_m^2 D_{Sm}\sigma^3)(D_{Pe}D_{Sm}+3D_{Pm})}{(L_m\sigma^2+D_{Sm})(3D_{Pm}-D_{Pe}L_m\sigma^2)(D_{Sm}S_0+L_m\sigma D_{Sm})^2} \quad (24)$$

where $B_R$ denotes the amperometric biosensor's sensitivity and $I(L_m)$ the steady-state biosensor current estimated at the enzyme layer thickness ($Lm$ ). Conductance is the inverse of resistance. Thus, this type of detection is known as a conductometric electrochemical biosensor.

## 4. RESULT AND DISCUSSION

The nonlinear equations (4) to (7) are solved analytically. Equations (14)- (17) are the new, simple and closed-form of analytical expressions for the concentration of substrate in enzyme layer $S_e(X)\ and$ in the enzyme membrane $S_m(X)$. $P_e(X)$ and $P_m(X)$ are concentration of product in enzyme layer and in the enzyme membrane for different values of parameters such as diffusion coefficients ($D_{Sm}, D_{pm}$ and $D_{pe}$), the maximal enzymatic rate ($v_{max}$), Michaelis constant ($k_M$) and diffusion module ($\sigma$).

### *4.1. Validation of analytical results with numerical simulation and previous results.*

In this section, the obtained approximate analytical expression for the steady-state concentrations and given by equations (14)-(17) are compared to numerical results obtained by the highly accurate MAPLE software. Our analytical expression of the concentration of substrates and products in the enzyme and membrane layer is compared with simulation results in Figures 2(a-d) for the experimental values of parameters. Baronas et al. [14] obtained analytical expression of current for the limiting cases only. In Figures 3(a-b), our current result is compared to the limiting case results of Baronal et al. [14] to demonstrate the efficiency of the current method. Figures S1-S4 are also provided in the supplementary materials (initiated with S) to compare concentrations with simulation results by using Maple software for one-compartment model ($Lm = 0$). It's mentioned that there's a good agreement.

When $L_m = 0$ the two-compartment model becomes one-compartment model. In this case analytical expressions of concentrations and current becomes as follows:

$$S_e(X) = \frac{S_0 \cosh\sqrt{\mu}X}{\cosh\sqrt{\mu}} \quad (25)$$

$$P_e(X) = \frac{S_0 D_{Sm}[X(D_{Pm}\cosh\sqrt{\mu}-4D_{Pm})-D_{Pm}\cosh\sqrt{\mu}X+4D_{Pm}]}{(D_{Pm})D_{Pe}D_{Sm}\cosh\sqrt{\mu}} \quad (26)$$

$$S_m(X) = \frac{S_0[(X-1)\sqrt{\mu}\sinh\sqrt{\mu}+D_{Sm}\cosh\sqrt{\mu}]}{D_{Sm}\cosh\sqrt{\mu}} \quad (27)$$

$$P_m(X) = \frac{S_0 D_{Sm}(1-X)[\sqrt{\mu}sinh(\sqrt{\mu})-\cosh(\sqrt{\mu})+4]}{(D_{Pm})(D_{Sm}\cosh\sqrt{\mu})} \quad (28)$$

where $\mu = \frac{\sigma^2}{1+S_e(0)}$ and $S_e(0) = \frac{S_0}{\cosh\sigma}$ .

The dimensionless current is,

$$I = \frac{il_e}{n_e F d S_e k_M} = \frac{S_0(\cosh\sqrt{\mu}-4))}{\cosh\sqrt{\mu})} \quad (29)$$

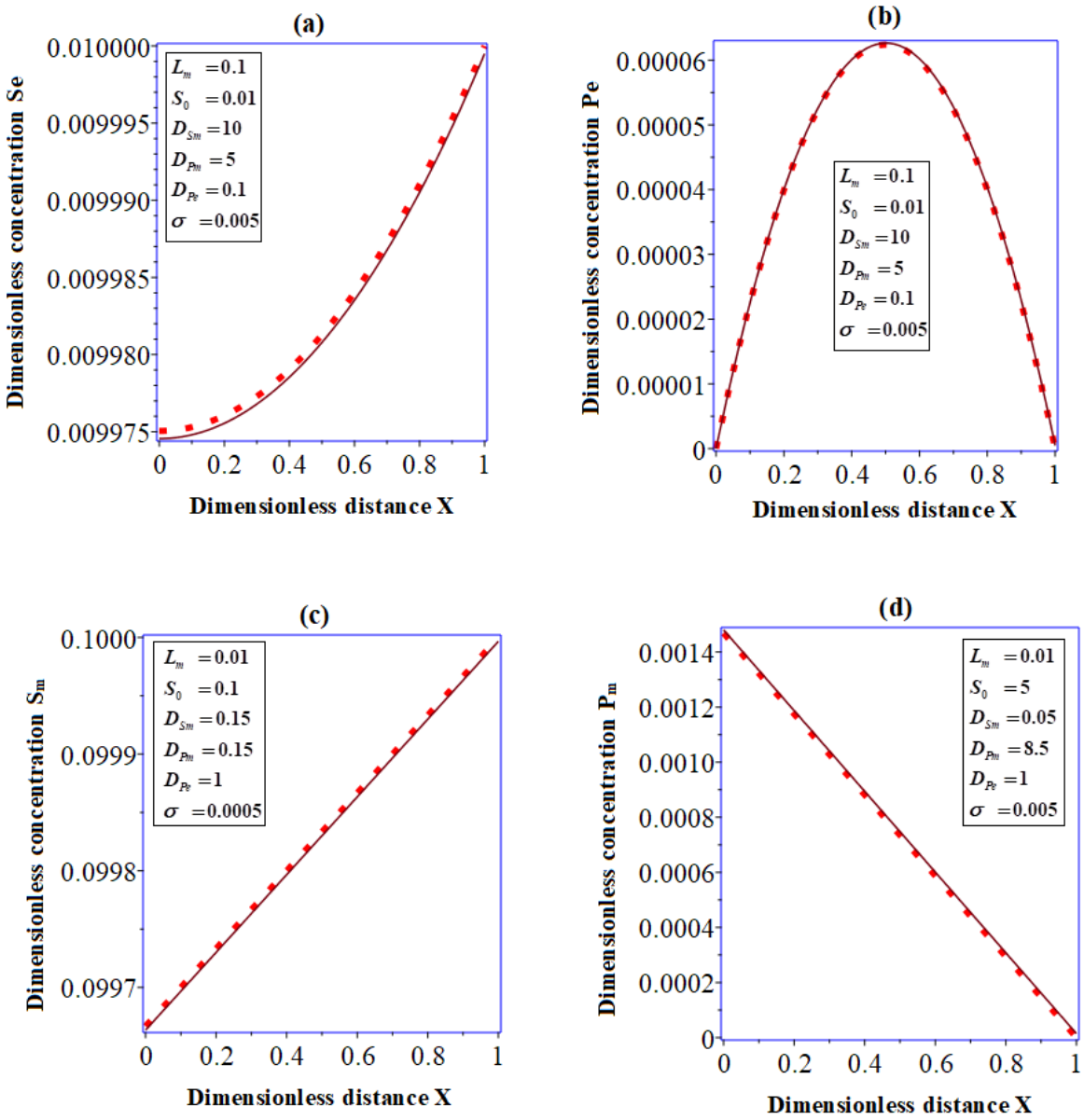


**Figure 2.** Dimensionless concentrations versus dimensionless distance. **(a)** Dimensionless concentration substrate in enzyme layer $Se(X)$. **b)** Dimensionless concentration product in the enzyme layer$Pe(X)$. **c)** Dimensionless concentration product in the membrane $Pm(X)$. **d)** Dimensionless concentration substrate in the membrane $Sm(X)$.

Figure.4a represents the concentration of substrate $S_e(X)$ versus normalized distance from the electrode surface for various values of parameter thicknesses ratio of enzyme layer and diffusion layer $(L_m)$ . In this figure, the concentration of the substrate increases when the parameter $L_m$ decreases. Figure.4b and Figure.4c shows the concentration of substrate versus normalized distance for various values of bulk concentration of substrate and Damköhler number. In those figures that the concentration of substrate increases when bulk concentration of substrate $(S_0)$ and diffusion module $(\sigma)$ increases. Figure 4d, illustrates the concentration of substrate for different values of parameter '$D_{Sm}$' . In this figure, the concentration increases with the decreasing of diffusion coefficients.

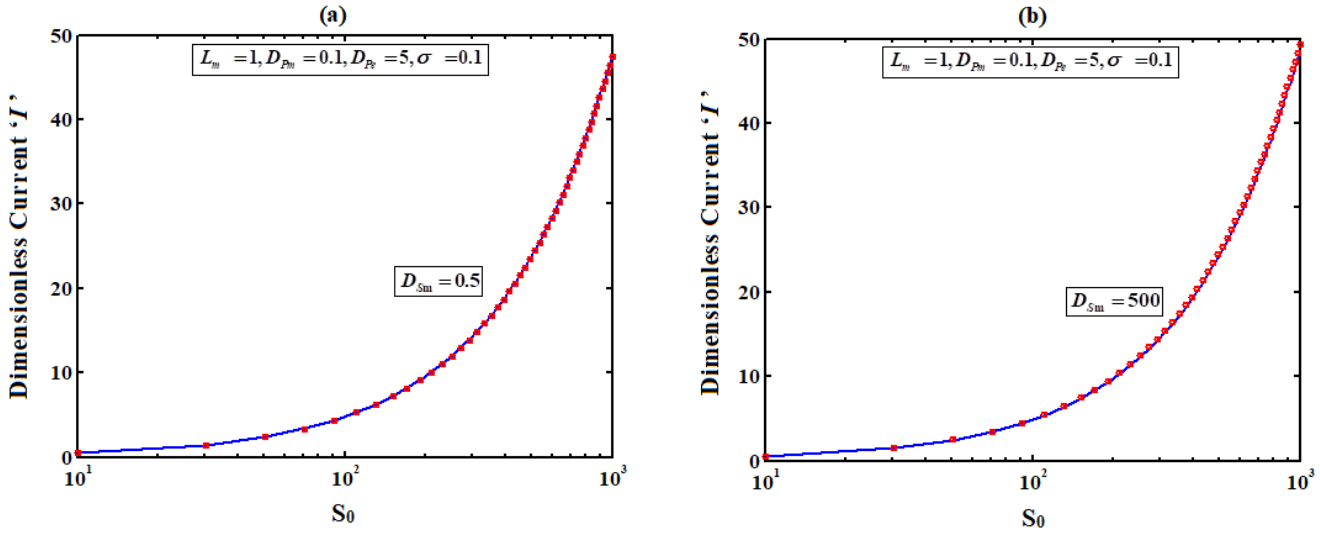


**Figure 3.** Comparison of dimensionless current ($I$) versus $S_0$ for various values of parameters with previous results[14]. Here bold line represents eqn. (19) and dotted line represents eqn. (21).

Figure.5a to Figure.5d, obtained the concentration of product ($P_e$) versus normalized distance for various values of thicknesses ratio of enzyme layer and diffusion layer and bulk concentration of substrate. It those figures that the concentration of product increases when $L_m, S_0, \sigma, D_{Sm}$ increases. Figure.5e inferred that the concentration of product versus normalized distance for various values of parameters. The figure represents the concentration of the product decreases when the diffusion coefficient ($D_{Pe}$) increases.

Figure.6a and Figure.6b shows the concentration of substrate versus normalized distance for various values of a parameter. In this figures that the concentration of the substrate increases when $S_0$ and diffusion coefficient ($D_{Sm}$) increases. Figure.6c denotes the concentration of substrate versus normalized distance for various values of parameter. It fallows that the concentration of substrate increases when parameter diffusion module ($\sigma$) decreases. Figure.7a to Figure.7c shows the concentration of products versus normalized distance with various values of the parameters. In those figures, we can see that the concentration increases when the parameters $\sigma, D_{\mathrm{Pm}}, D_{\mathrm{Pe}}$ decreases. The effect of the diffusion coefficient ($D_{Sm}$) and bulk concentration of substrate ($S_0$) on the concentration is exactly opposite to that of $\sigma, D_{\mathrm{Pm}}, D_{\mathrm{Pe}}$ as illustrated in Figure. 7d and Figure. 7e.

Figure 8, illustrates the effect of thicknesses of enzyme layer and diffusion layer ($L_m$ ) on sensitivity profile ($B_S$). In this figure, the values of parameters $L_m$ and bulk concentration of substrate ($S_0$) is increases when sensitivity is also increases.

The effect of different values of bulk concentration of substrate and ration of thicknesses of enzyme layer and diffusion layer for resistance profile is shown in Figure 9. It is observed that an increase in $S_0$ and $L_m$ leads to increase in resistance.

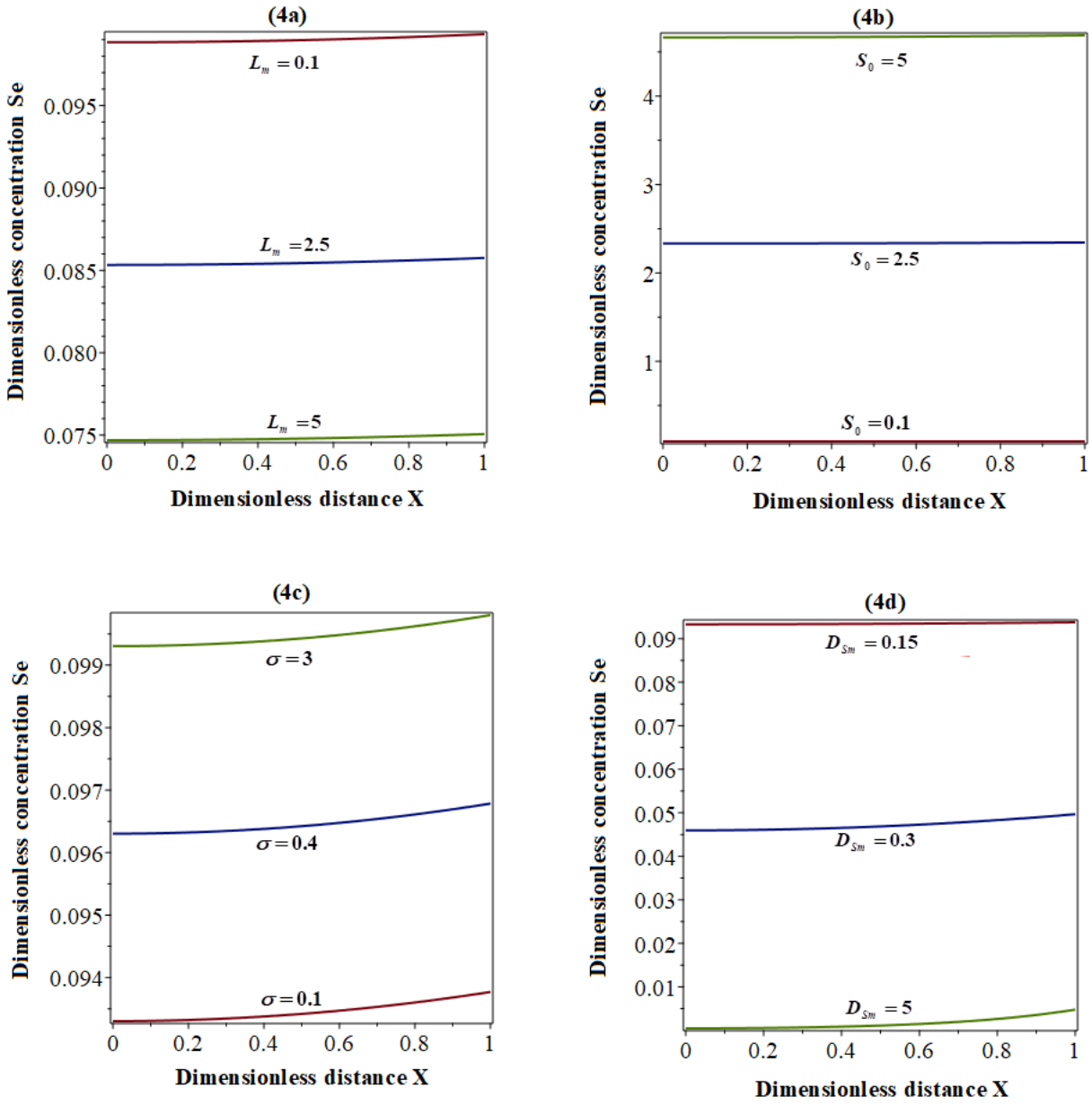


**Figure 4.** Dimensionless concentration of substrate $S_e(X)$, versus normalized distance $(X)$ for different values of parameters for some fixed experimental values of other parameters Common: $L_m = 1, S_0 = 0.1, \sigma = 0.1, D_{Sm} = 0.15, D_{\mathrm{Pm}} = 0.15, D_{\mathrm{Pe}} = 1$.

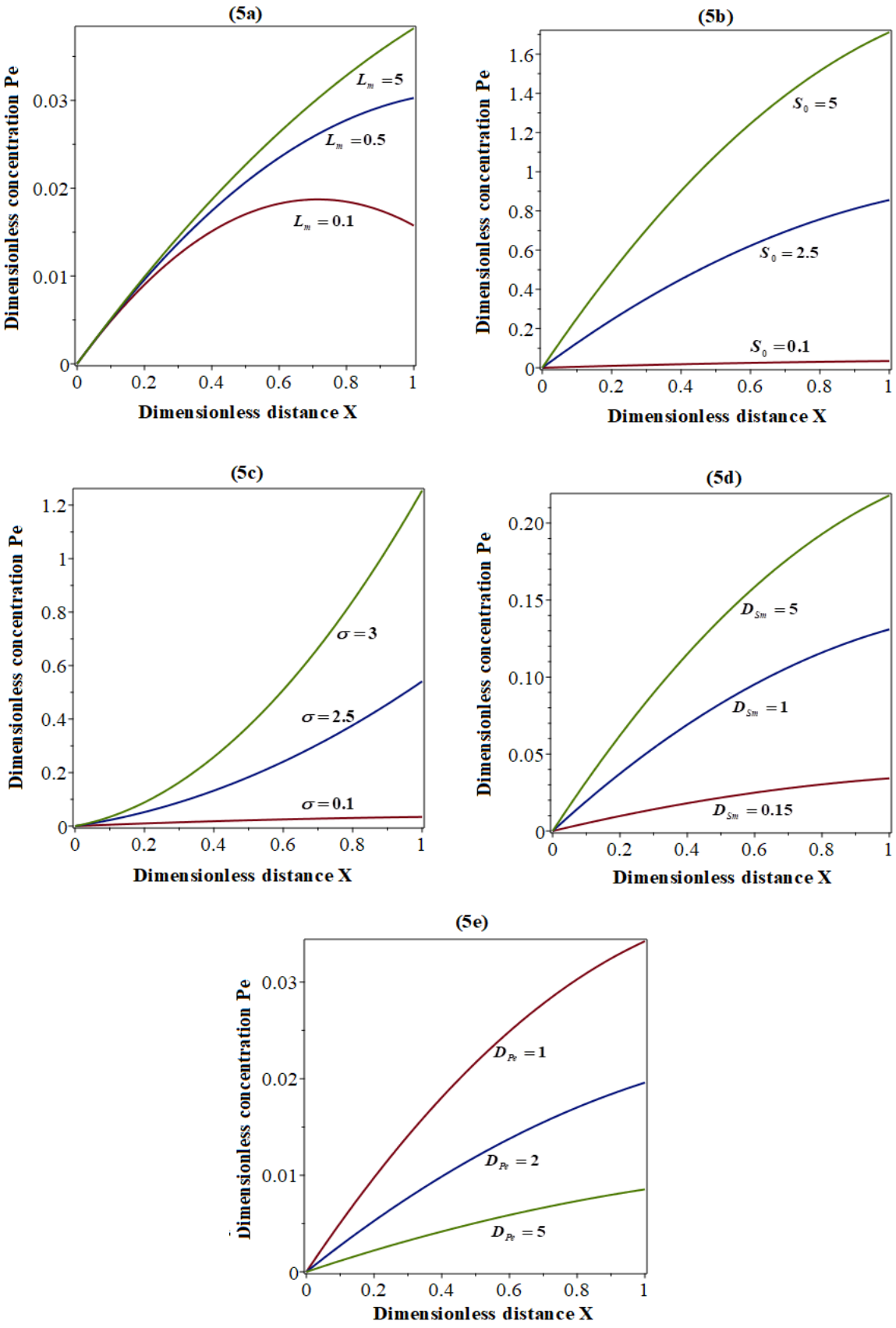


**Figure 5.** Dimensionless concentration of product $P_e(X)$, versus normalized distance $(X)$ for different values of parameters for some fixed experimental values of other parameters Common: $L_m = 1, S_0 = 0.1, \sigma = 0.1, D_{Sm} = 0.15, D_{\mathrm{Pm}} = 0.15, D_{\mathrm{Pe}} = 1.$

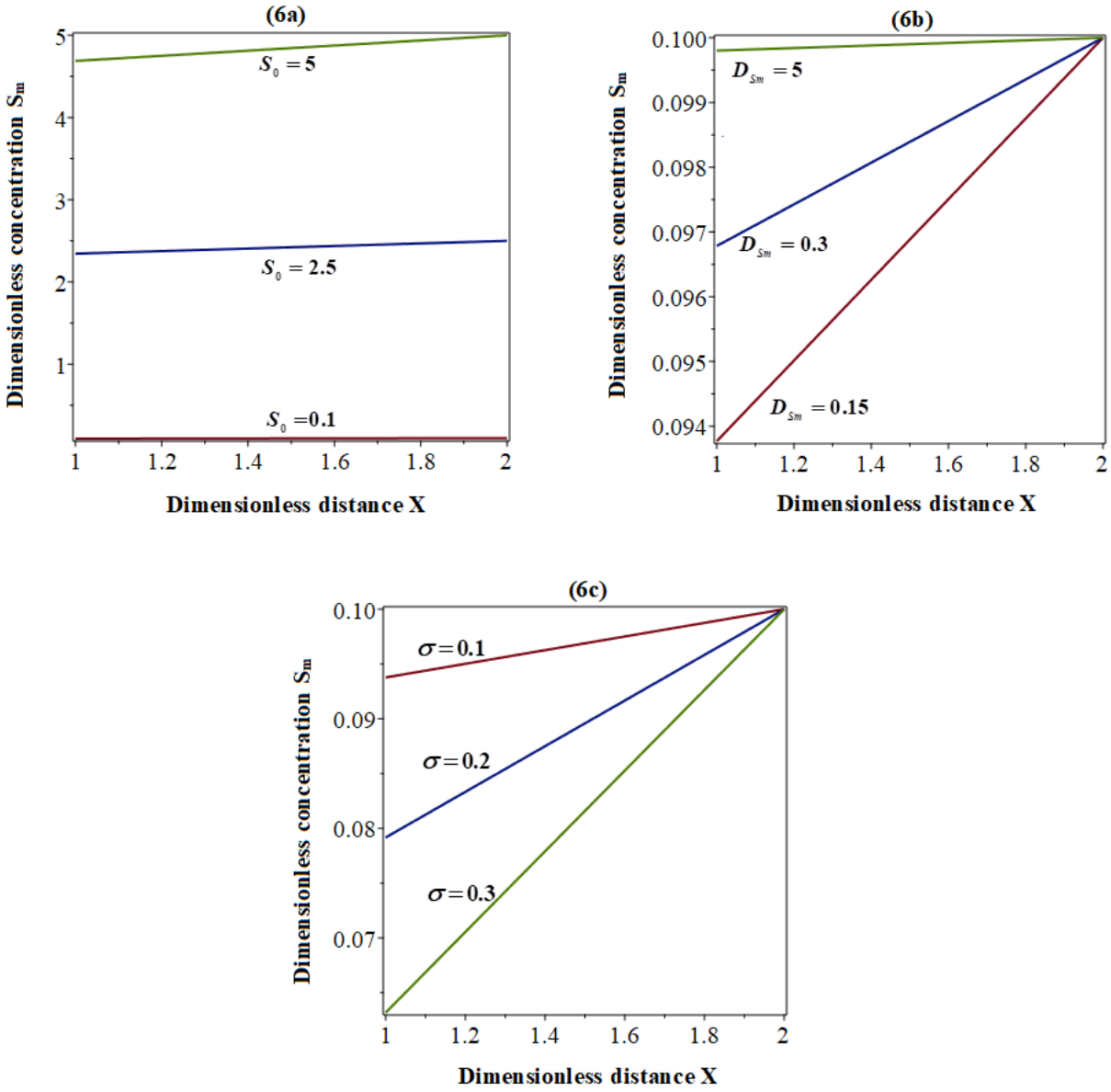


**Figure 6.** Dimensionless concentration of substrate $S_m(X)$, versus normalized distance $(X)$ for different values of parameters for some fixed experimental values of other parameters Common: $L_m = 1, S_0 = 0.1, \sigma = 0.1, D_{Sm} = 0.15, D_{\mathrm{Pm}} = 0.15, D_{\mathrm{Pe}} = 1.$

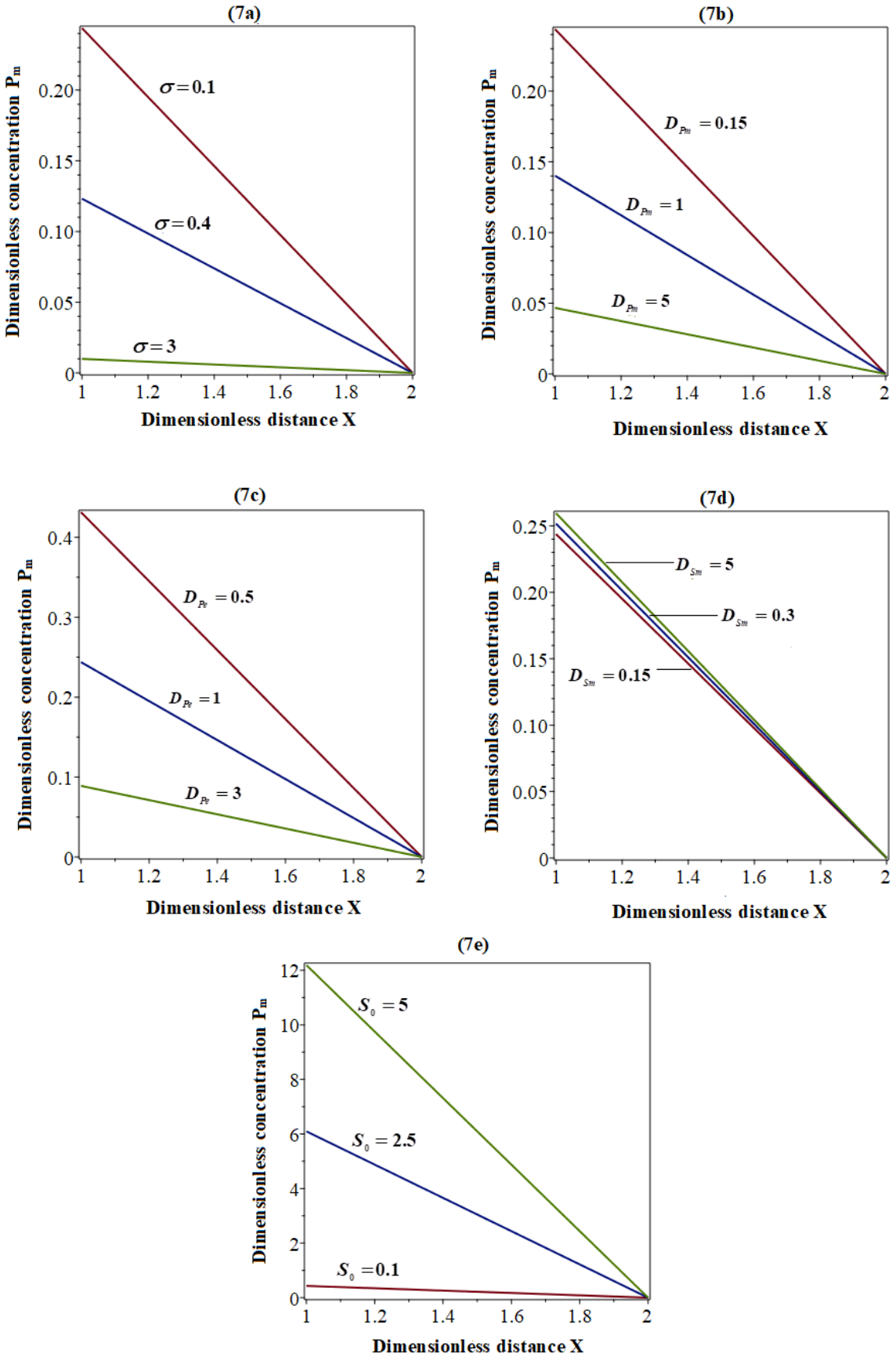


**Figure 7.** Dimensionless concentration of substrate $P_m(X)$, versus normalized distance $(X)$ for different values of parameters for some fixed experimental values of other parameters Common: $L_m = 1, S_0 = 0.1, \sigma = 0.1, D_{Sm} = 0.15, D_{\mathrm{Pm}} = 0.15, D_{\mathrm{Pe}} = 1.$

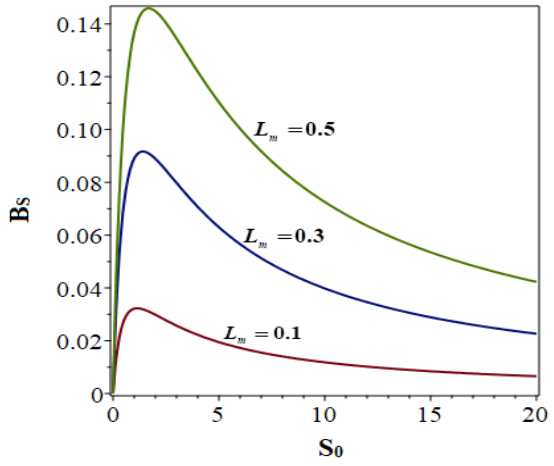


**Figure 8.** The sensitivity of biosensor using equation (23) for fixed values of $\sigma = 1, D_{Sm} = 1, D_{\mathrm{Pm}} = 1, D_{\mathrm{Pe}} = 1$ and various values of $L_m$.

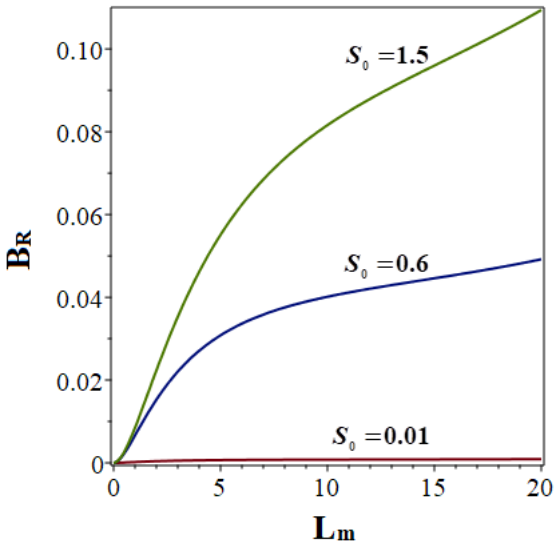


**Figure 9.** The resistance of biosensor using equation. (24) for fixed values of : $\sigma = 0.1, D_{Sm} = 0.15, D_{\mathrm{Pm}} = 0.15, D_{\mathrm{Pe}} = 1$ and various values of $S_0$.

## 4. CONCLUSIONS

This paper discusses the modelling of the enzyme-based two-compartment model of amperometric biosensors. The approximate analytical expression for the concentration of the substrates, products in the enzyme layer and enzyme membrane (diffusion layer) and current for all parameter values are obtained. The analytical results are compared with simulation results. The same method can be applied to find the concentration and current of the two-compartment model of potentiometric biosensor by including the zero-flux boundary conditions. These new analytical results provide a good understanding of the system and the enhancement of the parameters in the two-compartment model of the biosensor.

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NOMENCLATURE:

| **Parameters** | **Meanings** | **Units** |
|---|---|---|
| $s_e$ | Concentrations of the substrate in the enzyme layer | $\mu mol/cm^2$ |
| $p_e$ | Concentrations of the product in the enzyme layer | $\mu mol/cm^2$ |
| $s_m$ | Concentrations of the substrate in the enzyme membrane | $\mu mol/cm^2$ |
| $p_m$ | Concentrations of the product in the enzyme membrane | $\mu mol/cm^2$ |
| $S_e$ | Dimensionless concentrations of the substrate in the enzyme layer | None |
| $P_e$ | Dimensionless concentrations of the product in the enzyme layer | None |
| $S_m$ | Dimensionless concentrations of the substrate in the enzyme membrane | None |
| $P_m$ | Dimensionless concentrations of the product in the enzyme membrane | None |
| $d_{Se}, d_{Pe}, d_{Sm}, d_{Pm}$ | Diffusion coefficients | $\mu M^2/s$ |
| $D_{Se}, D_{Pe}, D_{Sm}, D_{Pm}$ | Dimensionless diffusion coefficients | None |
| $x$ | Distance from the electrode | $cm$ |
| $X$ | Dimensionless distance from the electrode | None |
| $v_{max}$ | Maximal enzymatic rate | $\mu M/s$ |
| $k_M$ | Michaelis -Menten C constant | $\mu M$ |
| $l_e$ | Thicknesses of the enzyme layer | $\mu m$ |
| $l_m$ | Thicknesses of the enzyme membrane | $\mu m$ |
| $L_m = \frac{l_m}{l_e}$ | Dimensionless thicknesses ratio of enzyme layer and diffusion layer | None |
| $I$ | Dimensionless current | None |

| | | |
|---|---|---|
| $i$ | Current density of the biosensor | $\mu A/cm^2$ |
| $S_0 = \frac{s_0}{k_m}$ | Dimensionless bulk concentration of substrate | None |
| $K_M^{app} = \frac{k_M^{app}}{k_M}$, | Dimensionless Michaelis-Menten constant | None |
| $n_e$ | Number of electrons involved in a charge transfer at the electrode surface | None |
| $F$ | Faraday constant (96485 C/mol) | C/mol |
| $B_s$ | Dimensionless sensitivity | None |
| $B_R$ | Dimensionless Resistance | None |
| $\sigma$ | Dimensionless diffusion module or the Damköhler number | None |

## APPENDIX – A: THE APPROXIMATE ANALYTICAL EXPRESSION NONLINEAR EQUATIONS (4) AND (5)

The homotopy for the eqns. (4) and (5) can be written as follows:

$$(1-p)\left[\frac{d^2S_e}{dX^2} - \frac{\sigma^2}{1+S_e(0)}S_e\right] - p\left[(1+S_e)\frac{d^2S_e}{dX^2} - \sigma^2 S_e\right] = 0 \quad \text{(A1)}$$

$$(1-p)\left[D_{Pe}\frac{d^2P_e}{dX^2} + \frac{\sigma^2}{1+S_e(0)}S_e\right] + p\left[(1+S_e)\frac{d^2P_e}{dX^2} + \sigma^2 S_e\right] = 0 \quad \text{(A2)}$$

where $S_e(0) = \frac{S_0 D_{Sm}}{L_m \sigma \sinh\sigma + D_{Sm}\cosh\sigma}$ (A3)

Now assume that approximate solution of the eqns. (A1) and (A2) as

$$S_e = S_{e0} + pS_{e1} + p^2 S_{e2} + \ldots \quad \text{(A4)}$$

$$P_e = P_{e0} + pP_{e1} + p^2 P_{e2} + \ldots \quad \text{(A5)}$$

Substituting the eqns.(A4) and (A5) in eqns. (A1) and (A2) and equating the coefficients of *p* on both sides we get,

$$p^o: \frac{d^2S_{e0}}{dX^2} - \frac{\sigma^2}{1+S_e(0)}S_{e0} = 0 \quad \text{(A6)}$$

$$p^0: D_{Pe}\frac{d^2P_{e0}}{dX^2} + \frac{\sigma^2}{1+S_e(0)}S_e = 0 \quad \text{(A7)}$$

The eqn. (A6) and (A7) can be written as

$$\frac{d^2S_{e0}}{dX^2} - \mu S_{e0} = 0 \quad \text{(A8)}$$

$$D_{Pe}\frac{d^2P_{e0}}{dX^2} + \mu S_{e0} = 0 \quad \text{(A9)}$$

where $\mu = \frac{\sigma^2}{1+S_e(0)}$ (A10)

The boundary conditions are

$$\left.\frac{dS_{e0}}{dX}\right|_{X=0} = 0, P_{e0}(0) = 0 \quad \text{(A11)}$$

$$S_m(1+L_m) = S_0, P_m(1+L_m) = 0 \quad \text{(A12)}$$

$$\left.\frac{dS_{e0}}{dX}\right|_{X=1} = D_{Sm}\left.\frac{dS_m}{dX}\right|_{X=1}, S_{e0}(1) = S_m(1) \quad \text{(A13)}$$

$$\left.\frac{dP_{e0}}{dX}\right|_{X=1} = \frac{D_{Pm}}{D_{Pe}}\left.\frac{dP_m}{dX}\right|_{X=1}, P_{e0}(1) = P_m(1) \quad \text{(A14)}$$

Solving the above equations using the boundary conditions we get

$$S_e(X) = \frac{S_0 D_{Sm}\cosh\sqrt{\mu}X}{L_m\sqrt{\mu}\sinh\sqrt{\mu}+D_{Sm}\cosh\sqrt{\mu}} \quad \text{(A15)}$$

$$P_e(X) = \frac{S_0 D_{Sm}\left[\frac{X(L_m D_{Pe}\sqrt{\mu}\sinh\sqrt{\mu}+D_{Pm}\cosh\sqrt{\mu}-4D_{Pm})}{L_m D_{Pe}+D_{Pm}} - (\cosh(\sqrt{\mu}X)-4)\right]}{D_{Pe}(L_m\sqrt{\mu}\sinh\sqrt{\mu}+D_{Sm}\cosh\sqrt{\mu})} \quad \text{(A16)}$$

$$S_m(X) = \frac{S_0((X-1)\sqrt{\mu}\sinh\sqrt{\mu}+D_{Sm}\cosh\sqrt{\mu})}{L_m\sqrt{\mu}\sinh\sqrt{\mu}+D_{Sm}\cosh\sqrt{\mu}} \quad \text{(A17)}$$

$$P_m(X) = \frac{S_0 D_{Sm}(1+L_m-X)[\sqrt{\mu}\sinh(\sqrt{\mu})-\cosh(\sqrt{\mu})+4]}{(L_m D_{Pe}+D_{Pm})(L_m\sqrt{\mu}\sinh\sqrt{\mu}+D_{Sm}\cosh\sqrt{\mu})} \quad \text{(A18)}$$

The dimensionless currents, '*I*' becomes

$$I = D_{Pe}\left.\frac{dP_e}{dX}\right|_{X=0} = \frac{il_e}{n_e F d S_e k_M} = \frac{S_0 D_{Sm}(L_m D_{Pe}\sqrt{\mu}\sinh\sqrt{\mu}+D_{Pm}(\cosh\sqrt{\mu}-4))}{D_{Pe}(L_m D_{Pe}+D_{Pm})(L_m\sqrt{\mu}\sinh\sqrt{\mu}+D_{Sm}\cosh\sqrt{\mu})} \quad \text{(A19)}$$

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SUPPLEMENTARY MATERIAL OF THE MANUSCRIPT.

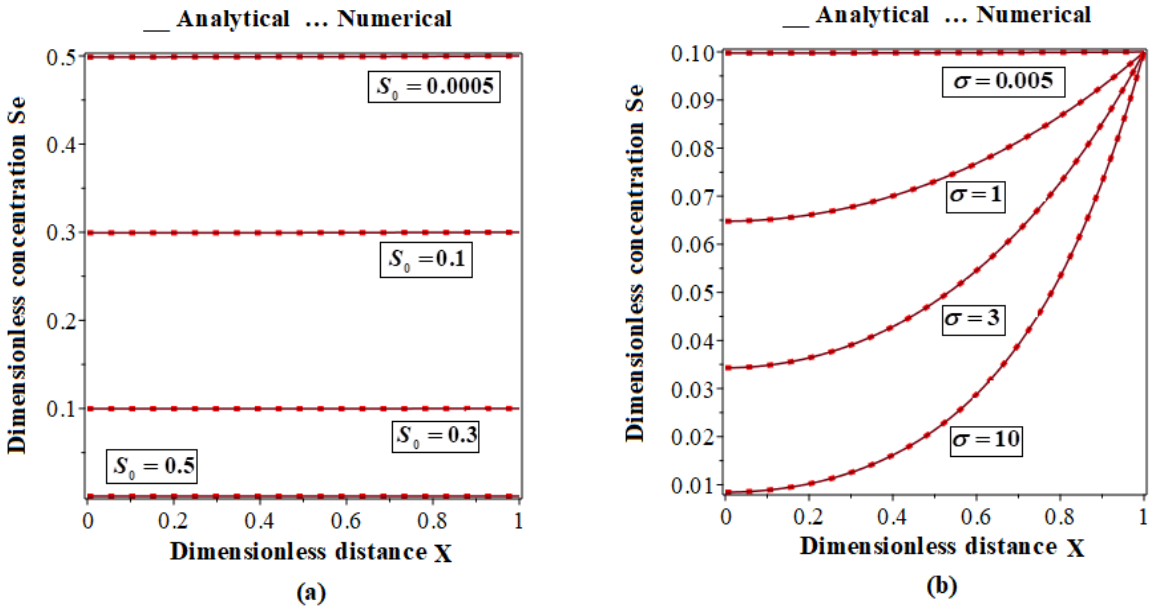


**Figure S1.** Dimensionless concentration of substrate $S_e(X)$, versus normalized distance $(X)$. (a) $S_0 = 0.0005, 0.1, 0.3, 0.5$ .( b) $\sigma = 0.005, 1, 3, 10$

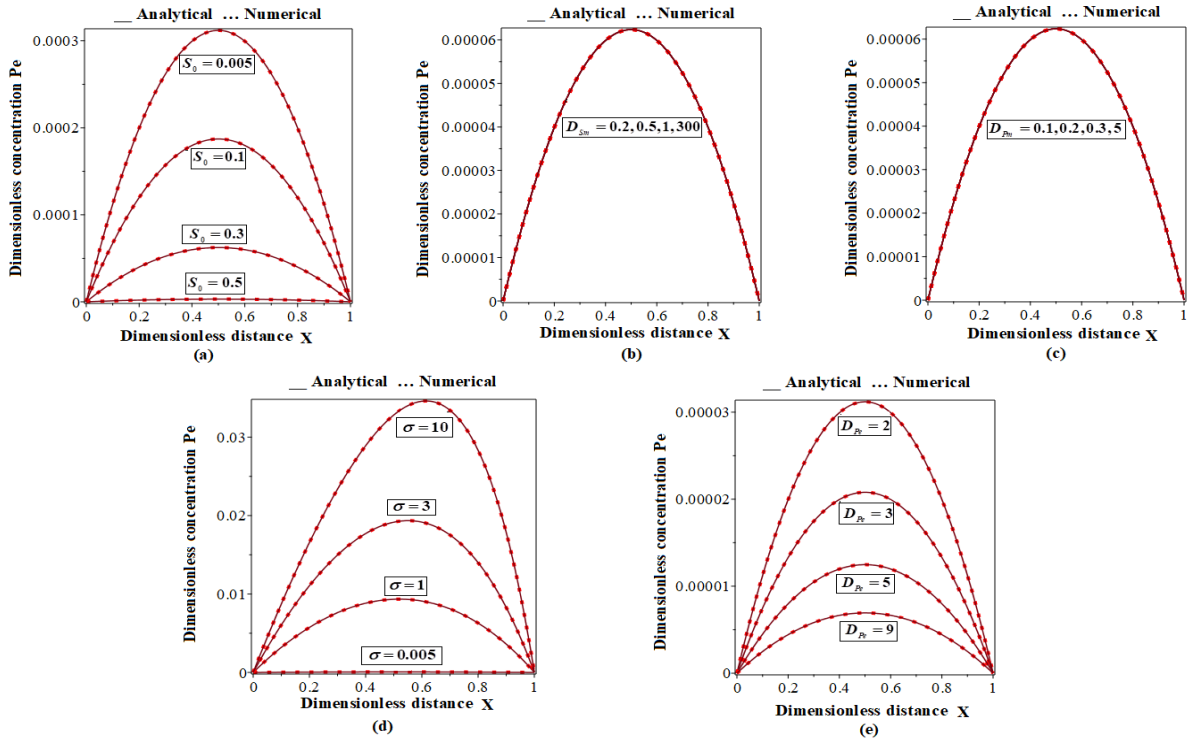


**Figure S2.** Dimensionless concentration of product $P_e(X)$, versus normalized distance $(X)$ (a) $S_0 = 0.005, 0.1, 0.3, 0.5$ ,( b) $D_{\mathrm{Sm}} = 0.2, 0.5, 1, 300$,(c) $D_{\mathrm{Pm}} = 0.1, 0.2, 0.3, 5$ (d) $\sigma = 0.005, 1, 3, 10$,(e) $D_{\mathrm{Pe}} = 2, 3, 5, 9$

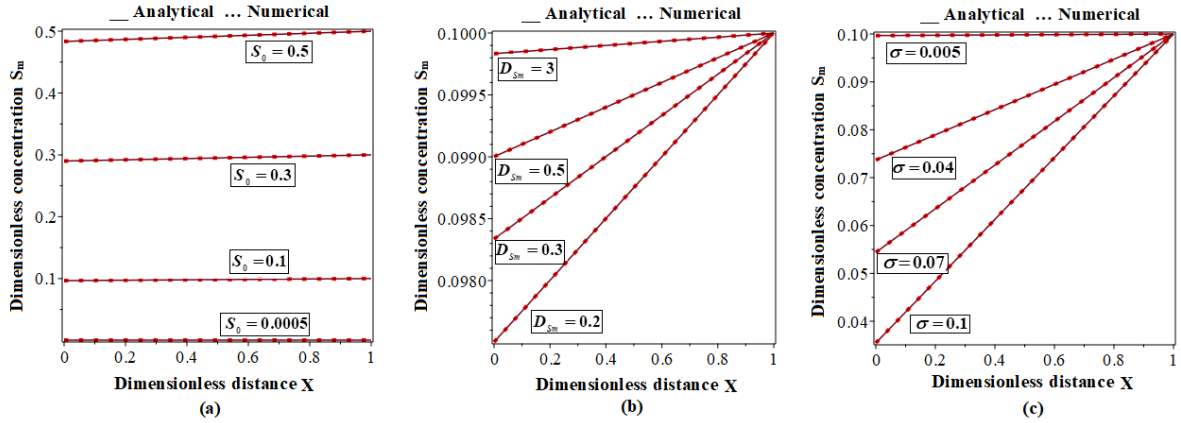


**Figure S3.** Dimensionless concentration of substrate $S_m(X)$, versus normalized distance $(X)$(a) $S_0 = 0.005, 0.1, 0.3, 0.5$ ,( b) $D_{\mathrm{Sm}} = 0.2, 0.3, 0.5, 3$,(c) $\sigma = 0.005, 0.04, 0.07, 0.1$

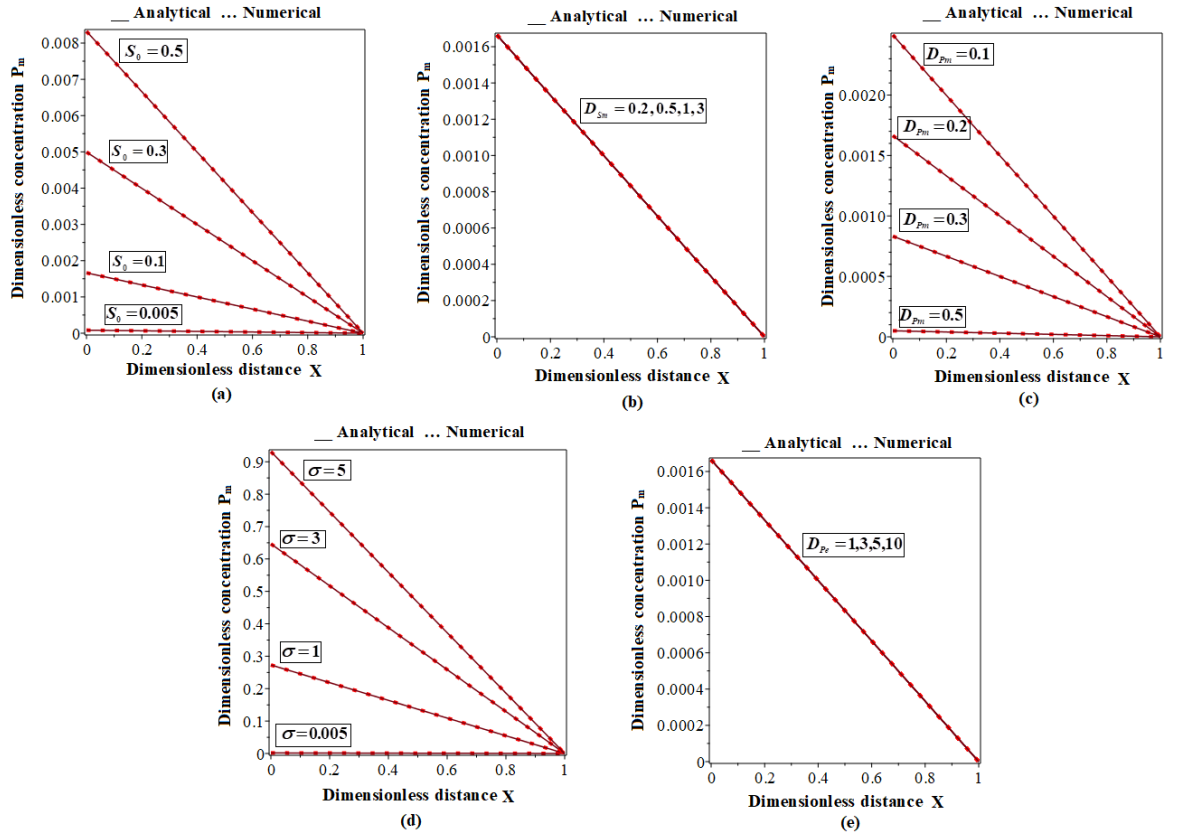


**Figure S4.** Dimensionless concentration of substrate$P_m(X)$, versus normalized distance $(X)$. (a) $S_0 = 0.005, 0.1, 0.3, 0.5$ ,( b) $D_{\mathrm{Sm}} = 0.2, 0.5, 1, 3$,(c) $D_{\mathrm{Pm}} = 0.1, 0.2, 0.3, 0.5$ (d) $\sigma = 0.005, 1, 3, 10$,(e) $D_{\mathrm{Pe}} = 1, 3, 5, 10$

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