

Review

Electroosmotic Stabilization on Soft Soil: Experimental Studies and Analytical Models (A historical review)

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Electrokinetic stabilization is a newly developed technique, which was pioneered by Cassagrand in 1940 and has been employed in railway embankment stabilization. Compared with conventional foundation treatment methods, electrokinetic stabilization performs more effectively with shorter times on soft clay foundations with a high content of fine grains and a low hydraulic coefficient. However, electroosmosis (EO) has not been widely used in practical engineering, especially in large-scale applications, due to a lack of understanding of the mechanism of EO treatment and the errors between the theoretical predictions and experimental results, which cannot be ignored. The purpose of this paper is to briefly review the history of electrokinetic stabilization on soft soils, including investigations into the mechanism, laboratory and field tests of the critical influencing factors, and the development of theoretical and numerical models and methods to improve the efficiency of EO treatment. In addition, directions for future research are discussed.

Keywords: Electrokinetic stabilization; Foundation treatment; Mechanism of EO treatment; Electrokinetic geosynthetics (EKG); Analytical and numerical solutions.

1. INTRODUCTION

Electroosmosis (EO) in soils is a complex process. When electrodes are inserted at opposite ends of a section of saturated soil and a dc voltage is applied on the electrodes, an electric field is generated in the soil. The free and film water will transfer from the anode to the cathode and drain out of the soil in relatively short time, leading to the consolidation of soils and ground improvement. According to Esrig's theory [1], a negative uneven pore water pressure will be generated due to the external electric field, so the effective stress in the soil will be positive throughout the soil, according

to the principle of effective stress. Then, surface settlement occurs, and the soil becomes consolidated under the effective stress.

Electroosmosis in soil was first observed by Russe in 1809, and since then, some scholars have studied different aspects of this technology. In 1939, Cassagrand [2] first applied the electroosmosis method to railway excavation engineering, and the electroosmosis method has been gradually applied to practical engineering applications. Conventional methods are usually subject to hydraulic permeability; for instance, surcharge preloading and vacuum preloading often require long treatment times and still cannot achieve the desired effect on some problematic soils. However, electroosmotic treatment is only relevant to the electroosmosis permeability coefficient but is irrelevant to the pore sizes in soil. Studies have indicated that the electroosmosis permeability coefficient values for most kind of soils are all approximately $5.0 \times 10^{-9} \text{ m}^2/\text{sV}$ [3]. As a result, this treatment is beneficial for soft or problematic soils. In recent years, due to increasing land reclamation projects, EO has drawn much attention and there have been many successful engineering applications. Kaniraj [4] treated humus soil with a high organic content and an initial moisture content of 305%~623%, while the moisture content decreased to 201~260% and the shear strength increased by approximately 600% after 6~8 days of EO treatment. Lo [5] proved that electroosmosis can permanently reduce the brittleness index and sensitivity of soil, improve soil quality, and obtain a more uniform treatment effect through electrode conversion technology. EO has been successfully used to repair the unstable slope of the British Stocks Green dam by J. Lamont-black [6] without closure to traffic. Electrokinetic geosynthetics, referred to as EKG, was adopted to dehydrate 400 m³ of nuclear waste for 1-5 days and successfully reduced the waste to 30 tons [7]. Compared with traditional methods such as adding inert materials to convert nuclear waste into a larger volume solid residue, the process cost is greatly reduced.

As a new foundation treatment method, electroosmosis still has many issues such as high energy consumption [8], a decrease in the effective potential due to the loss of contact between electrode and soil [9-10], anode corrosion leading to electrodes with shortened service life and secondary pollution of the soil [11] and inhomogeneous treatment effects [12]. The existing theories of electroosmotic consolidation lag behind the experimental research and are quite different from engineering practices because most models assume that the physical parameters of soil remain unchanged under small strain conditions. Hence, researchers have conducted a great deal of experiments and established modified theories, which aim to determine the external and internal factors that influence the electroosmosis efficiency and investigate the mechanism of the EO treatment.

This paper provides an overview of the experimental research and theoretical models of electroosmosis. The advantages of the applications of the EO method and its limitation are discussed, and several suggestions for further research are proposed.

2. ELECTRICAL PHENOMENON IN SOILS

There are usually four types of electrical phenomena when an electric field is applied to the soils: electrophoresis, ion migration, electroosmosis and electrolysis. Electrophoresis is the movement of charged soil particles, which is relative to the stationary liquid. Negative soil particles move from

cathodic area to anodic area, and as a result, the porosity of the anodic soil decreases and the strength increases. Ion migration is the movement of charged ions. The charged ions can move to the specific electrode under an electric force. Therefore, salt removal from soil can be achieved by applying an electric field to the soil. Electroosmosis and electrolysis have a great impact on soil improvement, and their performance will be discussed below.

2.1 Electroosmosis Flow

Electroosmosis is the process of the polarity water molecules including cationic ions moving from the high electric potential domains to the lower domains, which is relative to the stationary charged soil surface. In general, the velocity of the liquid phase in a porous medium can be expressed as follows,

$$J_i = \sum_{i=1}^n L_i X_i, \quad i = 1, 2, 3, \dots, \quad (1)$$

where J_i is the velocity of the flux, L_i stands for the conductivity coefficient and X_i is the driving force. The electroosmosis flow in soft soils can be written as,

$$Q = k_e i_e + k_h i_h \quad (2)$$

where k_e and k_h are electroosmotic and hydraulic permeability coefficients and i_e and i_h are electric potential and hydraulic gradients, respectively. Mitchell [18] compared the electroosmotic and hydraulic permeability coefficients of various soils, as shown in Table 1, and found that k_e is generally between $1 \times 10^{-9} \sim 1 \times 10^{-8} \text{ m}^2/\text{sV}$ and has a slight relationship with the type of soil. In addition, the electroosmosis coefficient for kaolin and bentonite was found to be $10^{-9} \text{ m}^2/\text{sV}$ by Jayasekera [19].

Table 1. Coefficients of Electroosmotic and Hydraulic permeability [18]

Soil type	Water Content (%)	k_e in $10^{-5} \text{ (cm}^2/\text{sV)}$	Approximate $k_h \text{ (cm/s)}$
London clay	52.3	5.8	10^{-8}
Boston blue clay	50.8	5.1	10^{-8}
Kaolin	67.7	5.7	10^{-7}
Clayer silt	31.7	5.0	10^{-6}
Rock flour	27.2	4.5	10^{-7}
Na-Montmorillonite	170	2.0	10^{-9}
Na-Montmorillonite	2000	12.0	10^{-8}
Mica powder	49.7	6.9	10^{-5}
Fine sand	26.0	4.1	10^{-4}
Quartz powder	23.5	4.3	10^{-4}
As quick clay	31.0	2.5-20.0	2.0×10^{-8}
Bootlegger Cove clay	30.0	2.4-5.0	2.0×10^{-8}
Silt clay, West Branch Dam	32.0	3.0-6.0	$1.2-6.5 \times 10^{-8}$
Clayer silt, Little Pic River, Ontario	26.0	1.5	2×10^{-8}

The Helmholtz-Smoluchowski (H-S) theory is widely accepted for the study of the electroosmosis coefficient. The theory presents the EO coefficient as follows,

$$k_e = \frac{n\varepsilon_w}{\mu} \zeta \quad (3)$$

In the equation above, n is the porosity of the soil, ε_w (F/m) is the permittivity of pore water, μ (N*s/m²) is the viscosity of pore water and ζ (V) is the zeta potential. The permittivity and viscosity of pore water can be considered constant in a very wide range; therefore, k_e is only related to the porosity of the soil and the zeta potential and is independent of the pore size from the equation above. These relationships are quite different from the hydraulic coefficient, which is strongly dependent on the pore size and distribution and varies between 10⁻⁴~10⁻⁹ cm/s for different soils. The H-S theory indicates that the EO process is more suitable for fine particle soil ground improvement than conventional methods. Based on the H-S theory, Shang [20] proposed a modified equation in which the curvature of the capillary is considered,

$$\frac{(k_e)_m}{n} = \alpha + \tau \frac{\varepsilon_w}{\mu} \zeta \quad (4)$$

where α and τ are the empirical coefficients, -2.84 and 0.0895, respectively. The coefficient τ stands for the influence of the curvature of the capillary channel on the electroosmosis coefficient. Table 2 gives the experimental data. In addition, the zeta potentials in Table 2 are absolute values and are negative.

Table 2. Zeta potential and k_e

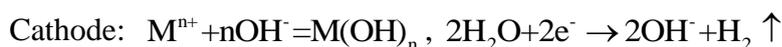
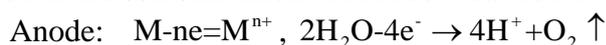
Soil	Zeta potential ζ [mV]	$(k_e)_p$, H-S model ($\times 10^{-9}$ m ² /(s·V))	$(k_e)_m$, measured ($\times 10^{-9}$ m ² /(s·V))
Grey	64	45	0.72
Brown	97	69	2.86
G-H	96	68	2.00
G-A	141	100	3.69
G-B	139	98	4.56
Phosphatic	62	44	0.7
Wallaceburg	87	62	1.5
Orleans	22	16	0
Kaolin-1	129	91	3.6
Kaolin-2	147	104	3.7

The above two equations imply that the electroosmosis coefficient is proportional to the zeta potential of the soil, and the higher the zeta potential is, the greater the electric permeability coefficient is. Studies show that the zeta potential is strongly affected by the pH value in soils since the charge property on the surface of the clay particles is related to the pH value of the solution. A zero net charge on the clay particles have been reported [21, 22] when the pH drops to a certain value. When the pH value is reduced continually, the negative charge on the surface of the clay particles can also be converted to a positive charge [22]. Under this condition, the zeta potential is positive, and the

electroosmosis flow will be from the cathode to the anode. In addition, the zeta potential is also affected by the mineral composition, solution type, ion concentration and their valence.

2.2 Electrolysis

Electrolysis is usually accompanied by a redox reaction of the metal electrode. Electrons shift between the metal electrode and liquid phase. The water molecules in the anodic regions lose electrons, and oxygen and H^+ ions are produced, forming an acid environment, while the water molecules in the cathodic regions obtain electrons, and hydrogen and OH^- ions are produced, forming an alkaline environment. Studies [23] have reported that part of the generated hydrogen and oxygen enter inside the soil during EO treatment, which contributes to the emergence of fissures and a reduction in the soil saturation. The redox reaction at the electrode are (M is the metal electrode):



Cheng [24] injected 1.5 mol $CaCl_2$ solution into the anode soil in a kaolin electroosmosis experiment, and a 2209 kPa bearing capacity of the soil near the cathode was observed after 120 h of EO treatment, while no significant improvement occurred in the anodic and central regions. The investigator believed that the free Ca^{2+} ions and OH^- ions demonstrate a pozzolanic reaction in an alkaline environment. The formatted calcium silicate and hydrated aluminate are the main reasons for the increase in the soil bearing capacity. Ou [25] pointed out that the invasion of hydration products into the soil pores also contribute to the improvement. The pozzolanic reaction are as follows:



The alkaline environment is much more suitable for ground improvement than an acidic environment. However, H^+ ions migrate approximately 1.8 times faster than OH^- ions, so the pH value of most of the soil is less than 7, which accelerates the corrosion rate of the anode and decreases the precipitation of the pore fluid near the anode. To prevent the diffusion of H^+ ions and expand the alkaline areas, Asavadorndeja [26] proposed an anode depolarization method, which is to add OH^- ions into the anode to neutralize the H^+ ions. With this method, the hydroxides can move backwards to the anode and increase the soil pH in anodic soils. The laboratory test results show that with this method, the improvement is more uniform, the shear strength of soil is increased by 500~600%, and the Atterberg limits are increased by 25%. Therefore, it is necessary to maintain the soil in an alkaline environment during the EO process so that more cement products could be generated, the soil strength could be improved more uniformly and the corrosion of the anode can be minimized.

3. FACTORS INFLUENCING THE EFFICIENCY

Electrokinetic stabilization is a complicated process that couples several physical fields including the stress-strain field, electric field, seepage field, thermal field, and concentration field. The

applications of electroosmotic stabilization are affected by many factors. Therefore, studying and reviewing the factors that affect the process is necessary in order to figure out the mechanism of electrokinetic stabilization. In summary, the existing studies have mainly focussed on the electrode materials and their configuration, the power condition and soil properties such as soil type, soil resistance, salt content and initial moisture content.

3.1 Electrode material and its configuration

Electrodes play a vital role in the electrokinetic stabilization of soft ground soil and should be inexpensive with a long service life and resist possible corrosion. Laboratory experimental results have shown that the efficiency of different electrode materials vary over a wide range, which is reflected in the effect of drainage consolidation and chemical reinforcement. In general, the electrodes can be divided into metal electrodes and non-metal electrodes. Table 3 lists the common materials and their effects in the experiments and practical engineering.

Table 3. Common electrode materials used from the literature.

Electrode materials	Authors	Advantages	Disadvantages
Copper	Tao[11],Burton[29] Lockhart[28]	Good conductivity, less loss in applied voltage.	Possible passivation, poor corrosion resistance, not economical in practice, cause soil contamination.
Aluminum	Tao[31], Zeng[45]	Economical efficient, good conductivity.	Easy to react with chemical agents, the generated aluminum -hydroxide cause voltage loss at late stages.
Stainless steel	Lockhart[28] Mohamedelhassan [47]	High corrosion resistance, high mechanical strength assures installation convenience.	Hard to shape
Carbon	Segall[30], Mohamedelhassan [47]	Hard to wear, high melting point, inert to most agents in soils.	High power loss at the electrodes and low energy efficiency, low mechanic strength
EKG	Glendinning[34], Kaniraj[35], Fourie[36]	Environmental friendly, recycle utilization, geotextile in EKG increase shear strength of soils.	Higher energy loss and longer duration time compared with metallic electrodes.

Shang [27] believes that the electrode should be evaluated according to electroosmosis efficiency, electrode corrosion and service life. The drainage velocity and treatment effects of the metal electrode are better than those of non-metallic electrode in common cases, but its disadvantages are also obvious. Metal electrodes are prone to oxidation in acidic environments, and the generated

metal ions could induce soil pollution. On the other hand, the metallic oxide will weaken the contact between soil and electrode surface, which leads to an increase in the electric resistance and reduces the energy efficiency. Field tests [28] have indicated that iron and copper electrodes exhibit a more significant effect than aluminium and graphite electrodes in kaolin. However, notably, the optimal electrode materials are usually different under disparate conditions. For example, Burton [29] compared the electroosmotic efficiency of graphite, iron, aluminium and copper electrodes and concluded that graphite and iron were superior to copper electrodes under a high potential gradient. In contrast, the results of Segall [30]. show that the average drainage rate of a graphite electrode is only approximately 50% that of an iron electrode. Tao's [31] work is consistent with Burton when the potential gradient is relatively high, but when it comes to a low gradient, the copper electrodes demonstrate the best performance.

In 1996, a newly developed material referred to as an EKG was first invented by Jones [32] and has attracted much attention since then. This material a polymer material that contains conductive material, which can minimize electrode corrosion. The electrode adopted in the literature [4] is named EVD (electric vertical drain) and consisted of wire, copper wire or conductive plastic, which is wrapped in a polymer material such as geotextile and filter cloths. Without electrode corrosion, the effective potential can maintain a high level, and the anode and cathode can be conversed freely, providing convenience when an electrode conversing technique is adopted. On the other hand, the reinforcement of the geotextile of the electrodes can also promote an increase in the shear strength of the soil [33]. The superiority of EKG allows its application on mine tailing and other high salt content soil dewatering. Successful engineering projects have been reported. Glendinning [12, 34] applied EKG to the dewatering of sewage sludge, the content of the solid particles increased by more than 30%, and the volume of the original sludge decreased by approximately 40% due to the high effective voltage. Kaniraj [35] observed a shear strength increase by a factor of 24 with the use of EKG. EKG materials are also adopted in the tailings dehydration process. Fourie [36] reported that the moisture content was reduced by 158% with only an energy consumption of 0.9 kWh/m^3 , which is completely acceptable in mine tailings dewatering projects. The EKG electrodes in Fourie's study are shown in Fig. 1, and the pore water pressure developed much faster than the normal method.



Figure 1. View of the electrokinetic geosynthetic material used in the Fourie [36] study.

The studies of the electrode configuration attempted to find the balance between the number of electrodes and the effect of electroosmosis. According to Alshwabkeh [37], the electric field area can be divided into an effective field and an invalid field, and the ratio between the effective and total area is the effective electric field ratio. The ratio can be increased by increasing the number of electrodes [38] and reducing the distance between the same polarity electrodes [40]. Indoor test results [12, 39] imply that hexagonal electrode arrangements are the most economically efficient configuration since their duration is the shortest, the increase in the shear strength is the largest and their average energy cost is between 39% and 53% lower than rectangular and staggered arrays. The reason is that [39] the hexagonal arrangements have the largest effective ratio (71%), larger than that of the rectangular (60%) and staggered arrangements (44%). Cassagrand [40] suggests that the distance between the anode and the cathode should be 1~3 m in practical projects. Although the existing studies give much information about the electrode arrangements, it is still difficult to determine the optimal method. However, influencing factors such as energy consumption, treatment effect, and electrode cost all have their own weight distribution. If the guidelines and consensuses are not available in advance, then it is difficult to judge the merits of the final result.

3.2 Power conditions

Normally, DC power is adopted during the electrokinetic stabilization, and the potential gradient is between 0.1~2.0 V/cm [41]. Shang [42] found that the unit of current at the interface of the soil, namely, the current density, dominates the electroosmosis effect. Under the same potential gradient, the greater the current density is, the larger the displacement is, and the shorter the processing time is. However, Shang [43] found that there is an upper limit for the voltage, and once the voltage is higher than this limit, the effect of electroosmosis no longer increases with applied voltage. Li [44] gave the optimal potential gradient of 1.25 V/cm for Hangzhou soft clay.

Researchers have studied the effects of different power modes on the effects of electroosmosis, such as the intermittent power, polarity reversal, and multistep power supply. Zeng [45] found that an intermittent power supply can increase the current density after the power is reconnected. Micic and Mohamedelhassan [46, 47] found that intermittent electricity can also reduce the degree of corrosion of the electrodes, and the average electric permeability coefficient is higher than that of continuous power, resulting in less energy consumption. Gong [48] found that the surface settlement, shear strength and moisture content is more uniform in an axisymmetric EO dewatering test using an intermittent current, which helps the postprocessing.

During electrokinetic stabilization, the anodic soils start to dry out as the water transfers to the cathode, which could lead to the loss of soil and electrode contact and a reduction in the current density. Polarity reversal is usually adopted to solve the problem. Shang [49] pointed out that the advantage of polarity reversal is to improve the evenness of electroosmotic treatment because the concentration of ions and the distribution of pH will tend to be uniform after the electrode is reversed. Asavadorndeja and Glawe [26] reported that this technique improved the shear strength by 570% through introducing hydroxyl ions to anode soils from cathode. Chen [50] carried out an indoor

experiment of polarity reversal under a one-dimensional condition, and pointed out that the shorter the reverse cycle is, the worse the effect of electroosmosis is. However, there is still debate about the effectiveness of polarity reversal. Polarity reversal can result in a more uniform treatment effect, but it will reduce the settlement of the soil and drained water with even higher energy consumption [51]. On one hand, the generated CSH and CAH at the cathode as well as the high consolidation soil near the anode make the drainage from the anode more difficult. On the other hand, the pore water near the original cathode is remigrated to the original anode, which can be seen as a waste of energy.

3.3 Soil electrical resistivity and conductivity

Soil conductivity is an important parameter and its unit is Siemens ($1\text{ S}=1\text{ A/V}$). The lower the soil resistance is, the larger the electric seepage is. Jones [52] points out that a suitable initial conductivity of the soil is between 0.005-0.5 S/m. Soil is three-phase object and there are many ways in which electrons move within the soil. Friedman and Corwin [53, 54] concluded that there are three types of transmission: the pore water, the connected solid and liquid phases, and the solid-liquid interface on the surface of the soil particles. When the moisture content is low, electrons move mainly through the connected solid and liquid phases. With an increase in the water content, the thickness of double layer will also become wider and provide a path for the electrons. When the water content exceeds the liquid limit, soil particles can be seen as ‘floating’ in the liquid phase, and the current conduction depends mainly on the liquid phase. The soil conductivity is affected by a variety of factors, such as soil type, moisture content, porosity, fluid conductivity, temperature, particle shape and salt content distribution, as well as the pore size. To study the performance of the soil resistivity, several models have been presented. For example, Archie [55] proposed a saturated cohesionless soil resistivity model; Waxman’s model [56] considered the solid and liquid phases as parallel; and Gong [57] predicted the cohesive soil resistance precisely considering the impact of the degree of saturation. A four-phase electrode method [58, 59] is usually used for the measurement of the soil conductivity in a Miller soil box. The resistance in electrokinetic stabilization is composed of three parts in series,

$$R_t = R_s + R_e + R_{e-s} \quad (5)$$

where R_t is the total resistance and R_s , R_e and R_{e-s} represent the resistance of the soil, the electrode and the interface of soil-electrode, respectively. At the late stage of the EO treatment, the resistance will increase rapidly. On one hand, the moisture content reduction leads to an increase; on the other hand, the poor contact at the soil-electrode interfaces play a more important role. Bjerrum [60] and Lo [61] presented that the effective potential only accounted for approximately 30% or less of the total potential, if no additional method was adopted to improve the contact. Hence, maintaining good contact between the electrode and soil is a problem that scholars pay considerable attention to. According to the model test by Wen [62], the effect of the size effect can be eliminated by appropriate surcharge load, which can avoid or postpone the generation of cracks, and the interface resistance can be reduced. Lefebvre [10] injected saline solutions into the anode at an early stage, making the soil-electrode contact favourable and the effective potential increased by approximately 100% compared to the untreated samples. Burnotte [63] employed special chemical methods to increase the conductivity

of the electrode and the interface, making the resistance smaller and improving the utilization efficiency of the electric energy. A method referred to as anode follow-up was proposed by Liu [64]. After the anodic soils dried out and the contact becomes poor, the anodes are pulled out then inserted again into the soil closer to cathodes. With this method, the voltage applied on the soil can be diminished due to the decrease in the distance between electrodes. Additionally, the contact between the interfaces can be improved, resulting in an impressive improvement in the energy efficiency.

3.4 Soil properties

The electroosmosis method is suitable for soil with a high moisture content, which can ensure the successful conduction of current in soil. The initial moisture content will not only affect the efficiency of the EO treatment but also determine the physical parameters such as the conductivity and pore distribution in soil. Table 4 includes the initial water content of soil in the literature. It can be seen that most of the moisture content of soil is above its own liquid limit, and a moisture content of approximately 1.4~1.7 times the liquid limit can give satisfactory results.

Table 4. Initial water content from the literature

Researcher	Type of the soil	Initial water content (%)	Liquid limit (%)
Lockhart [64]	Sand and coal washery tailings	104~334	Not mentioned
Kalumba [66]	Tunneling slurry	47.0	33.5
Lefebvre [10]	Clay	60~80	Not mentioned
Kaniraj [4]	Peat and Clayey silt	552 and 643	413 and 323
Alshawabkeh[67]	Boston blue clay	50	50
Bjerrum [8]	Norwegian quick clay	31	19
Shang [27]	Phosphate clay	413	141
Ou [25]	Kaolinite	60	46
Tao [39]	Hangzhou soft soil	80	45
Kaniraj [35]	Organic soil	221~254	245

The amount of salt content in the soil affects the zeta potential and EO permeability. The zeta potential is measurement of the strength of mutual exclusion or attraction among the particles, i.e., the smaller the molecules or dispersed particles are, the higher the absolute value of the zeta potential is and the more stable the system is. In other words, the dissolution or dispersion of the solute is more likely to happen instead of the aggregation of free ions when the zeta potential absolute value is high. As such, the higher the number of free cations at the solid-liquid interface in the soil is, the more easily the generated electric seepage is. When the level of salinity is high, despite the increasing conductivity of the liquid phase, the thickness of the double layer will decrease, and the zeta potential will be low. It is believed that electrokinetic stabilization is not suitable for soil with a high salinity level. Bergado [29] believes that the efficiency of electroosmosis is not ideal when the salt content exceeds 6,000 PPM (part per million), and Mitchell [16] believes that when the soil conductivity is higher than 2.5

mS/cm, it is not suitable for electroosmosis. In addition, the valence of the cations will also affect the zeta potential. The higher the valence is, the lower the zeta potential is.

4. ELECTROOSMOSIS CONSOLIDATION THEORIES AND NUMERICAL MODELS

Esrig [68] proposed a one-dimensional model for electroosmosis in soils and presented the following hypothesis: 1) the soil physical parameters such as hydraulic and electroosmotic conductivity remain unchanged during the process; 2) electrophoresis and possible electrochemical reactions can be negligible; 3) the applied potential gradient remains constant throughout the process of electroosmosis; 4) soil is isotropic and saturated during the whole process; 5) only the horizontal water flow is considered; 6) the water flow caused by the hydraulic gradients and potential gradients can be superimposed linearly; and 7) the control equations are under a small deformation condition. Based on the above assumptions, the governing equation under a one-dimensional condition is:

$$\frac{\partial^2 u}{\partial x^2} + \frac{k_e}{k_h} \gamma_w \frac{\partial^2 V}{\partial x^2} = m_v \frac{\partial u}{\partial t} \quad (6)$$

where u is the pore pressure, V is the applied voltage and γ_w is the unit weight of water. The boundary condition is the free drainage at the cathode, while the impervious boundary is at anode. The pore pressure and average degree of consolidation can be solved with method of the separation of variables, which is

$$u(x, t) = -\frac{k_e}{k_h} \gamma_w E(x) + \frac{2k_e \gamma_w E_M}{k_h \pi^2} \times \sum_{n=0}^{\infty} \frac{(-1)^n}{m^2} \sin\left(\frac{m\pi x}{l}\right) [\exp(-m^2 \pi^2 T_v)] \quad (7)$$

$$\bar{U} = 1 - \frac{4}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{m^3} e^{-m^2 T_v \pi^2} \quad (8)$$

In the equations, $E(x)$ stands for the potential at point x , E_M is the total voltage, n is an integer that starts at 0, $m = n + (l/2)$, and T_v is the time factor, $T_v = C_v t / l^2$. The maximum negative pore water pressure in the soil occurs at the anode, where $x = l$, and after a sufficient treatment time ($t \rightarrow \infty$), the pore pressure is,

$$u(x, \infty) = -\frac{k_e \gamma_w E(x)}{k_h x}, \quad u(l, \infty) = -\frac{k_e}{k_h} \gamma_w E \quad (9)$$

At this moment, the velocity of the water flow can be addressed as follows,

$$Q = k_e i_e + k_h i_h = k_e \frac{E(x)}{x} + \frac{k_h}{\gamma_w} (u + \gamma_w z) \quad (10)$$

Substituting the pore pressure u into Q , the value of Q is 0, indicating the end of the electrokinetic stabilization. After Esrig's theory, Wan and Mitchell proposed a developed model that considered the effects of loading and polarity reversal. The calculation results indicate that polarity reversal does facilitate a more uniform treatment. In 1998, Shang [70] proposed a two-dimensional model in which the horizontal and vertical hydraulic conductivities are equal. The governing equation is similar to the diffusion equation in form.

$$\frac{\partial \xi}{\partial t} = c_h \frac{\partial^2 \xi}{\partial x^2} + c_v \frac{\partial^2 \xi}{\partial y^2} \quad (11)$$

where ξ is a dummy variable, $\xi(x, y, t) = u(x, y, t) + E \gamma_w k_e x / k_h$, and c_h and c_v are the coefficient of

consolidation in the horizontal and vertical directions, respectively. The boundary conditions (drainage at the cathode, impervious at the anode) can be described by the dummy variable,

$$\xi(0, y, t) = 0 \text{ (Free drainage at the vertical drain and cathode)} \tag{12}$$

$$\xi(x, 0, t) = 0 \text{ (Free drainage at top of the soil)} \tag{13}$$

$$\frac{\partial \xi(L, y, t)}{\partial y} = 0 \text{ (Impervious boundary at the anode)} \tag{14}$$

$$\frac{\partial \xi(x, H, t)}{\partial y} = 0 \text{ (Impervious boundary at the bottom)} \tag{15}$$

When a load q is applied, the analytical solution can be addressed as follows,

$$\xi(x, y, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} E_{mn} \left\{ \sin \frac{(2m+1)\pi x}{2L} \exp \left[-T_x \frac{(2m+1)^2 \pi^2}{4} \right] \times \right. \tag{16}$$

$$\left. \sin \frac{(2n+1)\pi y}{2H} \exp \left[-T_y \frac{(2n+1)^2 \pi^2}{4} \right] \right\}$$

$$E_{mn} = \frac{16q}{\pi^2} \frac{1}{(2m+1)(2n+1)} + \frac{32k_e E \gamma_w L}{\pi^3 k_h} \frac{(-1)^n}{(2m+1)^2 (2n+1)} \tag{17}$$

where T_x and T_y are time factors in the x and y directions, respectively. Therefore, the pore pressure of any position at any time is known. The results of a numerical example show that 37% of soil reinforcement near the anode is dominated by electroosmosis, while the remaining 63% of soil improvement is induced by the external load. It is notable that the potential on the soil surface is assumed to be zero in the boundary conditions, which is inconsistent with the actual situation. Su [71] proved that the pore pressure caused by electroosmosis is only associated with the applied voltage and boundary conditions and is irrelevant to the initial conditions. The final pore pressure could be positive or negative, which is different from Esrig's theory. Lo [9] explained the cause of negative pore water pressure from another aspect. The author believes that electroosmosis water flow only occurs when there is an electric current in the soil, and the development of the pore pressure is dominated by the current density J and soil resistance p . The pore pressure with current density as the control factor can be written as,

$$U_e = -\frac{k_e}{k_h} \gamma_w J \int_0^x p(x) dx \tag{18}$$

Esrig's theory is derived based on a constant parameter assumption. With the EO treatment advancing, including but not limited to the hydraulic and electric permeability coefficient, the soil resistance and other parameters will change with time. To better simulate the EO process, researchers have proposed several modified models analytically or numerically.

Based on the fact that the velocities of the ions vary and the electric resistance exists at the interface, Zhuang [72] believed that net charges will accumulate near the electrodes, resulting in an alternation in the electric field. Therefore, by substituting the principle of current continuity with charge conservation, the differential equation of the charge accumulation model is built. The model suggests that the functional relationship between the potential gradient and time is the superposition of the linear function and trigonometric function. It is only when $t \rightarrow \infty$ that the electric potential meets the linear distribution assumed by Esrig. Li [43] fitted the experimental data and determined that the effective voltage is a linear decreasing function of time. He also concluded that when the water content decreases to the limit of the soil shrinkage, the contact area between the soil and electrode will not

change, and the effective potential will be stable. With Li's [43] work, Gong [73] proposed a modified model, and the difference between his work and Esrig's theory is proportional to the distance from the cathode. The farther the distance from the cathode is, the greater the difference is.

The void ratio e plays a vital role in the physical parameters of soils. Hu and Wu [15, 74] found a nonlinear relationship between e and hydraulic and electric conductivities, $k_h=0.00217 e^3/(1+e)$ and $\sigma_e=1.016 \times [e/(1+e)-0.349]$, respectively. On this basis, the two-dimensional governing equation that considers the change in the soil physical parameters [75] is derived.

$$\frac{\partial^2 u}{\partial z^2} + \frac{k_{ez}}{k_{hz}} \gamma_w \frac{\partial^2 V}{\partial z^2} = \frac{m_v \gamma_w}{k_{hz}} \frac{\partial u}{\partial t} = C_v \frac{\partial u}{\partial t} \frac{\partial}{\partial z} \left(\sigma_{ez} \frac{\partial V}{\partial z} \right) = 0 \quad (19)$$

where C_v is the consolidation coefficient, $m_v=1/(\lambda'+2G')$ is the coefficient of the volume compressibility, λ' is the elastic modulus and G' is the shear modulus under drainage conditions. The finite element method results indicate that the change in the soil physical parameters will significantly influence the development of the pore water pressure. When the parameters are assumed to be constant, the pore pressure develops faster than it realistically does. Therefore, the predicted duration should be lengthened if Esrig's theory is adopted in practical engineering. The water flow is simulated through a finite element program by V. Jeyakanthan [76], in which the soil elastoplasticity is considered. It has been pointed out that the theory of Esrig is only applicable to overconsolidated soil and when the potential gradient is relatively low.

Because electroosmosis in soils involves a coupling effect of multiple physical fields, the more influencing factors considered in the electroosmosis model, the more difficult it is to obtain the exact analytical solution. However, numerical solutions are relatively easy to obtain with the help of the finite element method (FEM). The effectiveness of the FEM is proven by an example in which the soil is under the condition of a large deformation with the piecewise linear method in the literature [77, 78]. It is believed that the surface settlement under a large deformation condition is smaller than that of the small deformation conditions. Yuan [79] suggested that the settlement difference between these two conditions can reach 28.5% when the value of k_{eo}/k_w is equal to 10.

Due to the discharge of pore water and the gas generated by an electrolytic reaction entering into the soil, the soil might be transformed from saturated to unsaturated. The theories and models mentioned above failed to evaluate the effect of the degree of saturation, which strongly impacts the stress-strain relationship of soil. Despite of the difficulty of bringing the theory of consolidation for unsaturated soil into consideration, there still have been attempts to investigate the impact. Gabrieli [80] summarized the following empirical formula for the permeability coefficient and saturation S_w :

$$k_{w,rel}=a_w(S_w)^{b_w}, k_{e,rel}=a_e(S_w)^{b_e}, k_{g,rel}=a_g(1-S_w)^{b_g}, \quad (20)$$

where $k_{w,rel}$, $k_{e,rel}$ and $k_{g,rel}$ are the relative hydraulic, electric and gas permeability coefficients, respectively, and a_w , a_g , a_e , b_w , b_e and b_g are the material parameters. Tamagnini [81] hypothesized that the gas pressure at the cathode during the whole process is atmospheric pressure and only the oxygen produced by the anode electrolysis is brought into the soil by the instantaneous current. The governing equation for the gas phase is derived,

$$-\mathbf{q}_g \cdot \mathbf{n} = \bar{q}_g = \bar{J}_e \frac{RT}{4Fp_g} \eta \quad (21)$$

where \bar{q}_g is the gas flowing in per unit area, \bar{J}_e is the instantaneous current density, R is the gas

constant, T stands for the absolute temperature and F is Faraday's constant. The parameter η is a positive number less than 1, which represents a loss in the electrodes. The surface settlement of numerical simulation and experimental data is illustrated in Fig. 2. The reason leads to the phenomenon is that gas trapped in the soil causes the soil stiffness to be larger, thus preventing settlement near the anode. A similar conclusion was drawn by Jiao [82], where the Barcelona basic model for unsaturated soil coupled with soil elastoplastic stress-strain relationship are taken into account. However, the empirical models only apply to a certain type of soil, and the acquisition of the empirical coefficient for other soils often needs considerable prerequisite work.

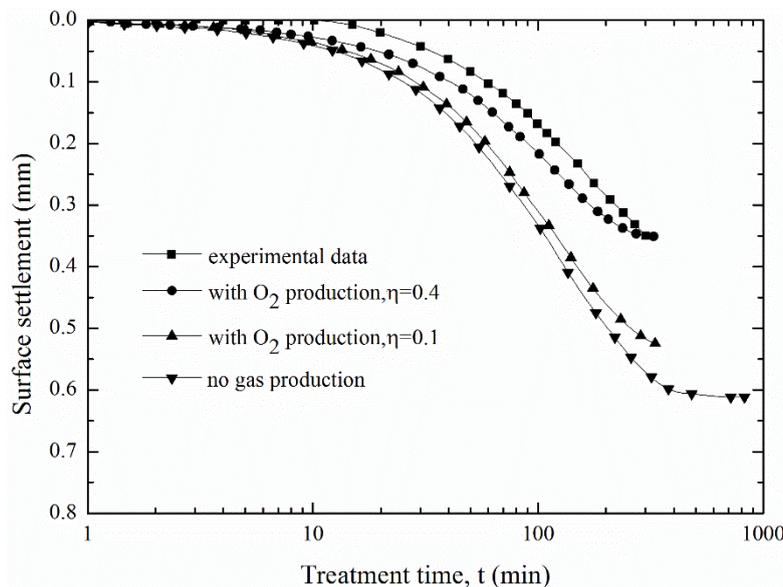


Figure 2. Surface settlement of the numerical simulation and experimental data drawn from Tamagnini [81].

The theories and numerical models for electroosmosis in soils can be classified into three major categories. The first category is represented by Esrig's [68] one-dimensional model whose main characteristic is that the physical parameters and external conditions are time independent, which is ideally simplified. However, these models are far from reality since many parameters are changing along with the electrokinetic stabilization process. To better reveal the mechanism, models and theories considering one or two parameters changing with time are proposed, for instance, Gong [73] and Hu and Wu's models [74]. The efforts of these authors mainly focus on diminishing the difference between the analytical results and experimental data. The third kind model couples the effects of the various physical fields, and it is usually hard to obtain their exact analytical solutions. As a result, FEM methods are adopted, especially when it involves unsaturated theories, such as in Tamagnini [81] and Jiao's [82]. The theoretical models of electroosmosis in soil have been developed for only a few decades and a comprehensive theory has not yet been demonstrated. The reasons are obvious, not only because the reinforcement mechanism of electroosmosis is not fully understood but also because of the complex multiphysical field coupling process involves too many factors, and changing an impact factor can lead to changes in multiple variables.

The effects of the electrochemical reaction and ion concentrations on the soil properties and electroosmotic water flow are rarely considered in existing theoretical models. Therefore, studies on the physiochemical changes during the EO treatment need to be done in the future.

5. METHODS IMPROVING EO EFFICIENCY

To improve the efficiency of electrokinetic stabilization and reduce energy consumption, other foundation treatment methods such as vacuum preloading or surcharge preloading, adding chemical additive, adopting inert electrodes (e.g., EKG, carbon electrode) [4, 12] are usually added to avoid possible electrode corrosion, as previously stated. Wang [83] believed that the initial application of the EO method could effectively accelerate the drainage process after the moisture content is reduced to the optimal threshold of 85% by vacuum preloading. The poor treatment of deep soil in vacuum preloading can be fixed by combining this method with EO treatment. Sun [84] pointed out that the electric power causes horizontal seepage, while vacuum pressure leads to vertical drainage. It is suggested that vacuum preloading and electroosmosis should be used in sequence, and their alternation time can be determined by the drainage amount. The principle of surcharge preloading is similar to that of vacuum preloading. Yu [85] conducted indoor experiments, and compared with single surcharge preloading, a 180~238% increase in strength was observed when EO was combined with surcharge preloading. In addition, the successful applications of electroosmosis combined with pneumatic fracturing [86] or low-energy dynamic compaction [87] have also been reported.

6. CONCLUSION

In this paper, the experimental results and theoretical progress of the electroosmotic strengthening of soft soil are summarized as follows: 1) the soil properties change during the electroosmosis process; and 2) the analysis of the mechanism of electroosmosis, such as the influence of the electrode material and loading condition; 3) the establishment and solutions of the theoretical and numerical models; 4) the methods to improve the efficiency of electroosmosis, such as a combination of other methods and electrochemical modifications; and 5) applications of new materials in electroosmotic treatments are discussed in detail.

Although researchers have performed considerable work, electrokinetic stabilization in soils is still far from being fully understood. Soil properties change with time and this change leads to a difference in the predicted and actual results. In addition, chemical reactions occur near the electrodes, resulting in possible acidification or alkalization of the soils. The influence of these reactions on the conductivity of the electrodes and soil properties are hard to describe quantitatively. The author suggests that additional investigations should include the following aspects: 1) field and laboratory tests should focus on soil composition, physicochemical and engineering characteristics; 2) improved computing methods to accurately calculate the displacement, energy consumption, and strength development; 3) electroosmosis performance on special soils with advanced materials (e.g., EKG, high

salinity soil); and 4) the establishment of the theoretical models of multiphysical fields during the entire process, including studies of chemical solutions and unsaturated soil performance.

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